

Freescale Semiconductor Application Note

Document Number: AN3666 Rev. 0, 11/2010

Software Optimization of FFTs and IFFTs Using the SC3850 Core

The Fast Fourier Transform (FFT) is a numerically efficient algorithm used to compute the Discrete Fourier Transform (DFT). The Radix-2 and Radix-4 algorithms are used mostly for practical applications due to their simple structures. Compared with Radix-2 FFT, Radix-4 FFT provides a 25% savings in multipliers. For a complex N-point Fourier transform, the Radix-4 FFT reduces the number of complex multiplications from N² to $3(N/4)\log_4N$ and the number of complex additions from N² to $8(N/4)\log_4N$, where \log_4N is the number of stages and N/4 is the number of butterflies in each stage. FFTs are of importance to a wide variety of applications, such as telecommunications (3GPP-LTE, WiMAX, and so on). For example, Orthogonal Frequency Division Multiplexing (OFDM) signals are generated using the FFT algorithm.

This application note describes the implementation of the Radix-4 decimation-in-time (DIT) FFT algorithm using the Freescale StarCore SC3850 core. The document discusses how to use new features available in the SC3850 core, such as dual-multiplier, to improve the performance of the FFT. Code optimization and performance results are also investigated in this document. Typical reference code is included in this document to demonstrate the implementation details.

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1 Introduction

1.1 Overview

The discrete Fourier transform (DFT) plays an important role in the analysis, design, and implementation of discrete-time signal processing algorithms and systems because efficient algorithms exist for the computation of the DFT. These efficient algorithms are called Fast Fourier Transform (FFT) algorithms. In terms of multiplications and additions, the FFT algorithms can be orders of magnitude more efficient than competing algorithms.

It is well known that the DFT takes N² complex multiplications and N² complex additions for complex N-point transform. Thus, direct computation of the DFT is inefficient. The basic idea of the FFT algorithm is to break up an N-point DFT transform into successive smaller and smaller transforms known as butterflies (basic computational elements). The small transforms used can be 2-point DFTs known as Radix-2, 4-point DFTs known as Radix-4, or other points. A two-point butterfly requires 1 complex multiplication and 2 complex additions, and a 4-point butterfly requires 3 complex multiplications and 8 complex additions. Therefore, the Radix-2 FFT reduces the complexity of a N-point DFT down to (N/2)log₂N complex multiplications and Nlog₂N complex additions since there are log₂N stages and each stage has N/2 2-point butterflies. For the Radix-4 FFT, there are log₄N stages and each stage has N/4 4-point butterflies. Thus, the total number of complex multiplication is (3N/4)log₄N = (3N/8)log₂N and the number of required complex additions is 8(N/4)log₄N = Nlog₂N.

Above all, the radix-4 FFT requires only 75% as many complex multiplies as the radix-2 FFT, although it uses the same number of complex additions. These additional savings make it a widely-used FFT algorithm. Thus, we would like to use Radix-4 FFT if the number of points is power of 4. However, if the number of points is power of 2 but not power of 4, the Radix-2 algorithm must be used to complete the whole FFT process. In this application note, we will only discuss Radix-4 FFT algorithm.

Now, let's consider an example to demonstrate how FFTs are used in real applications. In the 3GPP-LTE (Long Term Evolution), M-point DFT and Inverse DFT (IDFT) are used to convert the signal between frequency domain and time domain. 3GPP-LTE aims to provide for an uplink speed of up to 50Mbps and a downlink speed of up to 100Mbps. For this purpose, 3GPP-LTE physical layer uses Orthogonal Frequency Division Multiple Access (OFDMA) on the downlink and Single Carrier - Frequency Division Multiple Access (SC-FDMA) on the uplink. Figure 1 shows the transmitter and receiver structure of OFDMA and SC-FDMA systems.

We can see from this example that DFT and IDFT are the key elements to represent changing signals in time and frequency domains. In real applications, FFTs are normally used to for high performance instead of direct DFT calculation.



Figure 1. Transmitter and Receiver Structure of SC-FDMA and OFDMA Systems

1.2 Organization

The rest of the document is organized as follows:

- Section 2, "Radix-4 FFT Algorithm" gives a brief background for DFT, and describes (decimation in frequency) DIF and (decimation in time) DIT Radix-4 FFT algorithms.
- Section 3, "SC3850 Data Types and Instructions" investigates some features in SC3850, which can be used for efficient FFT implementation.
- Section 4, "Implementation on the SC3850 Core" provides the detailed implementation on SC3850, and discusses fixed-point implementation issues. How to fully utilize the resource in SC3850 and optimize the implementation is also discussed. Source code is included for reference.
- Section 5, "Experimental Results" presents experimental results.
- Section 6, "Conclusions" presents the conclusion.
- Section 7, "References" provides a list of useful references.



Radix-4 FFT Algorithm

2 Radix-4 FFT Algorithm

2.1 DFT and IDFT

The Fast Fourier Transform (FFT) is a computationally efficient algorithm to calculate a Discrete Fourier Transform (DFT). The DFT X(k), k=0,1,2,...,N-1 of a sequence x(n), n=0,1,2,...,N-1 is defined as

$$\begin{split} X(k) &= \sum_{\substack{n = 0 \\ N-1}} x(n) exp\left(-j\frac{2\pi nk}{N}\right) & \textit{Eqn. 1} \\ &= \sum_{\substack{n = 0 \\ n = 0}} x(n) W_N^{nk} & \textit{W}_N^{nk} \\ W_N^{nk} &= exp\left(-j\frac{2\pi nk}{N}\right) & \textit{Eqn. 2} \\ &= \cos\left(\frac{2\pi nk}{N}\right) - j\sin\left(\frac{2\pi nk}{N}\right) \end{split}$$

In Equation 1 and Equation 2, N is the number of data, $j = \sqrt{-1}$, and W_N^{nk} is the twiddle factor. Equation 1 is called the N-point DFT of the sequence of x(n). For each value of k, the value of X(k) represents the Fourier transform at the frequency $\frac{2\pi k}{N}$. The IDFT is defined as follows:

$$\begin{aligned} \mathbf{x}(\mathbf{n}) &= \frac{1}{N} \sum_{\substack{k=0 \\ N-1}}^{N-1} \mathbf{X}(k) \exp\left(j\frac{2\pi nk}{N}\right) & \text{Eqn. 3} \\ &= \frac{1}{N} \sum_{\substack{k=0 \\ k=0}}^{N-1} \mathbf{X}(k) \mathbf{W}_{N}^{-nk} \\ \mathbf{W}_{N}^{-nk} &= \exp\left(j\frac{2\pi nk}{N}\right) \\ &= \cos\left(\frac{2\pi nk}{N}\right) + j\sin\left(\frac{2\pi nk}{N}\right) \end{aligned}$$

Equation 3 is essentially the same as Equation 1. The differences are that the exponent of the twiddle factor in Equation 3 is the negative of the one in Equation 1 and the scaling factor is 1/N. The IDFT can be simply computed using the same algorithms for DFT but with conjugated twiddle factors. Alternatively, we can use the same twiddles factors for DFT with conjugated input and output to compute IDFT. Equation 1 is also called the *analysis equation* and Equation 3 the *synthesis equation*.



From Equation 1, it is clear that to compute X(k) for each k, it requires N complex multiplications and N-1 complex additions. So, for N values of k, that is, for the entire DFT, it requires N² complex multiplications and $N(N-1) \approx N^2$ complex additions. Thus, the DFT is very computationally intensive. Note that a multiplication of two complex numbers $(a+jb) \times (c+jd) = (ac-bd) + j(bc+ad)$ requires four real multiplications and two real additions. A complex addition (a+jb) + (c+jd) = (a+c) + j(b+d) requires two real additions.

We will present two commonly used FFT algorithms: decimation in frequency (DIF) and decimation in time (DIT). Please note that the Radix-4 algorithms work out only when the FFT length N is a power of four.

2.2 Radix-4 DIF FFT

We will use the properties shown by Equation 5 in the derivation of the algorithm.

Symmetry property:
$$W_N^{k+\frac{N}{2}} = -W_N^k$$
Eqn. 5Periodicity property: $W_N^{k+N} = W_N^k$

The Radix-4 DIF FFT algorithm breaks a N-point DFT calculation into a number of 4-point DFTs (4-point butterflies). Compared with direct computation of N-point DFT, 4-point butterfly calculation requires much less operations. The Radix-4 DIF FFT can be derived as shown in Equation 6.

$$\begin{split} X(k) &= \sum_{n=0}^{n} x(n) W_{N}^{nk} \\ &= \sum_{n=0}^{n=0} x(n) W_{N}^{nk} \\ &= \sum_{n=0}^{n} x(n) W_{N}^{nk} + \sum_{n=\frac{N}{4}} x(n) W_{N}^{nk} + \sum_{n=\frac{2N}{4}} x(n) W_{N}^{nk} + \sum_{n=\frac{3N}{4}} x(n) W_{N}^{nk} \\ &= \sum_{n=0}^{\frac{N}{4}-1} x(n) W_{N}^{nk} + \sum_{n=0}^{n} x\left(n + \frac{N}{4}\right) W_{N}^{\left(n + \frac{N}{4}\right)k} + \sum_{n=0}^{\frac{N}{4}-1} x\left(n + \frac{2N}{4}\right) W_{N}^{\left(n + \frac{2N}{4}\right)k} + \sum_{n=0}^{\frac{N}{4}-1} x\left(n + \frac{3N}{4}\right) W_{N}^{\left(n + \frac{3N}{4}\right)k} \\ &= \sum_{n=0}^{\frac{N}{4}-1} x(n) W_{N}^{nk} + W_{N}^{\frac{N}{4}} \sum_{n=0}^{\frac{N}{4}-1} x\left(n + \frac{N}{4}\right) W_{N}^{nk} + W_{N}^{\frac{2Nk}{4}} \sum_{n=0}^{\frac{N}{4}-1} x\left(n + \frac{3N}{4}\right) W_{N}^{nk} \\ &= \sum_{n=0}^{\frac{N}{4}-1} x(n) W_{N}^{nk} + W_{N}^{\frac{Nk}{4}} \sum_{n=0}^{\frac{N}{4}-1} x\left(n + \frac{N}{4}\right) W_{N}^{nk} + W_{N}^{\frac{2Nk}{4}} \sum_{n=0}^{\frac{N}{4}-1} x\left(n + \frac{3N}{4}\right) W_{N}^{nk} \\ &= \sum_{n=0}^{\frac{N}{4}-1} \left\{ x(n) + W_{N}^{\frac{Nk}{4}} x\left(n + \frac{N}{4}\right) + W_{N}^{\frac{2Nk}{4}} x\left(n + \frac{2N}{4}\right) + W_{N}^{\frac{3Nk}{4}} x\left(n + \frac{3N}{4}\right) \right\} W_{N}^{nk} \\ &= \sum_{n=0}^{\frac{N}{4}-1} \left\{ x(n) + W_{N}^{\frac{Nk}{4}} x\left(n + \frac{N}{4}\right) + W_{N}^{\frac{2Nk}{4}} x\left(n + \frac{2N}{4}\right) + W_{N}^{\frac{3Nk}{4}} x\left(n + \frac{3N}{4}\right) \right\} W_{N}^{nk} \\ &= 0, 1, 2, 3, ..., N - 1 \end{split}$$

Radix-4 FFT Algorithm

The special factors of $W_N^{\frac{Nk}{4}}$, $W_N^{\frac{2NK}{4}}$, and $W_N^{\frac{3Nk}{4}}$ in above equation can be calculated as shown in Equation 7.

$$W_{N}^{\frac{Nk}{4}} = \exp\left(-j\frac{2\pi k}{N} \cdot \frac{N}{4}\right) = \exp\left(-j\frac{\pi k}{2}\right) = (-j)^{k}$$

$$W_{N}^{\frac{2Nk}{4}} = \exp\left(-j\frac{2\pi k}{N} \cdot \frac{2N}{4}\right) = \exp(-j\pi k) = (-1)^{k}$$

$$Eqn. 7$$

$$W_{N}^{\frac{3Nk}{4}} = \exp\left(-j\frac{2\pi k}{N} \cdot \frac{3N}{4}\right) = \exp\left(-j\frac{3\pi k}{2}\right) = j^{k}$$

Then, Equation 6 can be rewritten as shown in Equation 8.

$$X(k) = \sum_{n=0}^{\frac{N}{4}-1} \left\{ x(n) + (-j)^k x \left(n + \frac{N}{4} \right) + (-1)^k x \left(n + \frac{2N}{4} \right) + j^k x \left(n + \frac{3N}{4} \right) \right\} W_N^{nk}$$

$$k = 0, 1, 2, 3, ..., N - 1$$

$$Eqn. 8$$

Considering $\{x(n) + (-j)^k x(n + N/4) + (-1)^k x(n + (2N)/4) + j^k x(n + (3N)/4)\}\$ as one signal, Equation 8 looks very similar to a N/4-point FFT. However, it is not an FFT of length N/4 because the twiddle factor W_N^{nk} depends on N instead of N/4. To make this equation an N/4-point FFT, the transform X(k) can be broken into four parts as shown in Equation 9.

$$\begin{split} X(4k) &= \sum_{n=0}^{\frac{N}{4}-1} \left\{ x(n) + x\left(n + \frac{N}{4}\right) + x\left(n + \frac{2N}{4}\right) + x\left(n + \frac{3N}{4}\right) \right\} W_{N}^{0} W_{N/4}^{nk} \\ X(4k+1) &= \sum_{n=0}^{\frac{N}{4}-1} \left\{ x(n) - jx\left(n + \frac{N}{4}\right) - x\left(n + \frac{2N}{4}\right) + jx\left(n + \frac{3N}{4}\right) \right\} W_{N}^{n} W_{N/4}^{nk} \\ X(4k+2) &= \sum_{n=0}^{\frac{N}{4}-1} \left\{ x(n) - x\left(n + \frac{N}{4}\right) + x\left(n + \frac{2N}{4}\right) - x\left(n + \frac{3N}{4}\right) \right\} W_{N}^{2n} W_{N/4}^{nk} \\ X(4k+3) &= \sum_{n=0}^{\frac{N}{4}-1} \left\{ x(n) + jx\left(n + \frac{N}{4}\right) - x\left(n + \frac{2N}{4}\right) - jx\left(n + \frac{3N}{4}\right) \right\} W_{N}^{3n} W_{N/4}^{nk} \\ k &= 0, 1, 2, 3, ..., \frac{N}{4} - 1 \end{split}$$

In Equation 9, the twiddle factors are obtained as shown in Equation 10.

$$\begin{split} W_{N}^{4nk} &= \exp\left(-j\left(\frac{2\pi \cdot 4nk}{N}\right)\right) = \exp\left(-j\frac{2\pi nk}{N/4}\right) = W_{N}^{0}W_{N/4}^{nk} \\ W_{N}^{n(4k+1)} &= \exp\left(-j\left(\frac{2\pi \cdot n(4k+1)}{N}\right)\right) = \exp\left(-j\frac{2\pi n}{N} - j\frac{2\pi nk}{N/4}\right) = W_{N}^{n}W_{N/4}^{nk} \\ W_{N}^{n(4k+2)} &= \exp\left(-j\left(\frac{2\pi \cdot n(4k+2)}{N}\right)\right) = \exp\left(-j\frac{2\pi \cdot 2n}{N} - j\frac{2\pi nk}{N/4}\right) = W_{N}^{2n}W_{N/4}^{nk} \end{split}$$

$$\begin{split} \textbf{Eqn. 10} \\ W_{N}^{n(4k+3)} &= \exp\left(-j\left(\frac{2\pi \cdot n(4k+3)}{N}\right)\right) = \exp\left(-j\frac{2\pi \cdot 3n}{N} - j\frac{2\pi nk}{N/4}\right) = W_{N}^{3n}W_{N/4}^{nk} \end{split}$$

If we use the definitions shown in Equation 11.

$$\begin{split} y(n) &= \left\{ x(n) + x\left(n + \frac{N}{4}\right) + x\left(n + \frac{2N}{4}\right) + x\left(n + \frac{3N}{4}\right) \right\} W_N^0 \\ y(n + N/4) &= \left\{ x(n) - jx\left(n + \frac{N}{4}\right) - x\left(n + \frac{2N}{4}\right) + jx\left(n + \frac{3N}{4}\right) \right\} W_N^n \end{split}$$

$$\begin{aligned} & \textit{Eqn. 11} \\ y(n + (2N)/4) &= \left\{ x(n) - x\left(n + \frac{N}{4}\right) + x\left(n + \frac{2N}{4}\right) - x\left(n + \frac{3N}{4}\right) \right\} W_N^{2n} \end{aligned}$$

From Equation 9, we can see that X(4k), X(4k+1), X(4k+2), and X(4k+3) are N/4-point FFT of y(n), y(n+N/4), y(n+2n/4), and y(n+3N/4), respectively. As a result, an N-point DFT is reduced to the computation of four N/4-point DFTs.

Figure 2 shows the corresponding Radix-4 butterfly calculation of Equation 11. Through further rearrangement, it can be shown that each Radix-4 DIF butterfly requires of 3 complex multiplications and 8 complex additions (12 real multiplications and 22 real additions). The decimation of the data sequences referred to as stage 1 of the decomposition. This process can be repeated again and again until the resulting sequences are reduced to one-point sequences. For $N=4^p$, this decimation can be performed $p=\log_4 N$ times. An example is shown in Figure 4 to illustrate the whole process. Thus the total number of complex multiplications is $3(N/4)\log_4 N$ since there are 3 complex multiplications each butterfly, N/4 butterflies each stage, and $\log_4 N$ stages. The number of complex additions is $8(N/4)\log_4 N$ since there are 8 complex additions each butterfly, N/4 butterflies each stage, and $\log_4 N$ stages.



Radix-4 FFT Algorithm



Figure 2. Radix-4 DIF Butterfly

In Figure 2, each of output y(n), y(n + N / 4), y(n + 2N / 4), and y(n + 3N / 4) is a sum of four input signals x(n), x(n + N / 4), x(n + 2N / 4), and x(n + 3N / 4), each multiplied by either +1, -1, j, or -j, and then the sum is multiplied by a twiddle factor $(1, W_N^n, W_N^{2n}, W_N^{3n})$. The process of multiplication after addition is called "Decimation In Frequency (DIF)". On the other hand, the process of addition after the multiplication is called "Decimation In Time (DIT)". To illustrate the algorithm clearly, a simplified butterfly is shown in Figure 3.



Figure 3. Simplified Radix-4 DIF Butterfly

The Radix-4 DIF FFT algorithm processes an array of data by successive passes over the input data samples. On each pass, the algorithm performs Radix-4 DIF butterflies, where each butterfly picks up four complex data and returns four complex data as shown in Figure 2 and Figure 3. The input data, output data, and the twiddle factors are assumed in the following format:

Input data:

Output data:

$$y(n) = Ar' + j \times Ai'$$

$$y\left(n + \frac{N}{4}\right) = Br' + j \times Bi'$$

$$y\left(n + \frac{2N}{4}\right) = Cr' + j \times Ci'$$

$$y\left(n + \frac{3N}{4}\right) = Dr' + j \times Di'$$
Eqn. 13

Twiddle factors:

$$\begin{split} W_N^n &= Wbr + j \times Wbi \\ W_N^{2n} &= Wcr + j \times Wci \\ W_N^{3n} &= Wdr + j \times Wdi \end{split} \end{tabular} \end{tabular} \end{tabular} \end{tabular}$$

The real and imaginary part of output data for Radix-4 DIF butterfly is specified in Equation 15.

$$\begin{array}{l} Ar' = Ar + Br + Cr + Dr \\ = (Ar + Br) + (Cr + Dr) \\ Ai' = Ai + Bi + Ci + Di \\ = (Ai + Ci) + (Bi + Di) \\ Br' = (Ar + Bi - Cr - Di) \times Wbr - (Ai - Br - Ci + Dr) \times Wbi \\ = ((Ar - Cr) + (Bi - Di)) \times Wbr - ((Ai - Ci) - (Br - Dr)) \times Wbi \\ Bi' = (Ai - Br - Ci + Dr) \times Wbr + (Ar + Bi - Cr - Di) \times Wbi \\ = ((Ai - Ci) - (Br - Dr)) \times Wbr + ((Ar - Cr) + (Bi - Di)) \times Wbi \\ Cr' = (Ar - Br + Cr - Dr) \times Wcr - (Ai - Bi + Ci - Di) \times Wci \\ = ((Ar + Cr) - (Br + Dr)) \times Wcr - ((Ai + Ci) - (Bi + Di)) \times Wci \\ Ci' = (Ai - Bi + Ci - Di) \times Wcr + (Ar - Br + Cr - Dr) \times Wci \\ = ((Ai + Ci) - (Bi + Di)) \times Wcr + ((Ar + Cr) - (Br + Dr)) \times Wci \\ Dr' = (Ar - Bi - Cr + Di) \times Wdr - (Ai + Br - Ci - Dr) \times Wdi \\ = ((Ar - Cr) - (Bi - Di)) \times Wdr + ((Ar - Bi - Cr + Di)) \times Wdi \\ Di' = (Ai + Br - Ci - Dr) \times Wdr + (Ar - Bi - Cr + Di) \times Wdi \\ = ((Ai - Ci) + (Br - Dr)) \times Wdr + ((Ar - Cr) - (Bi - Di)) \times Wdi \\ \end{array}$$



Radix-4 FFT Algorithm

As we can see from Equation 15 that there are 12 real multiplications and 22 real additions (count only once for duplicated additions and multiplications). It is equivalent to 3 complex multiplications and 8 complex additions since one complex multiplication requires four real multiplications plus two real additions and one complex addition requires two real additions. As mentioned before, in Radix-4 DIF butterfly algorithm, the N-point FFT consists of $\log_4(N)$ stages, and each stage consists of N/4-point Radix-4 DIF butterflies. Therefore, Radix-4 DIF butterfly calculation reduces the number of complex multiplications). For example, the number of real multiplications needed for a 1024-point DFT is reduced from $4N^2 = 4194304$ to $3N\log_4N = 15360$. The improvement of the Radix-4 DIF butterfly algorithm over the direct calculation of the DFT is approximately 273 times.

An example of 16-point Radix-4 DIF FFT diagram is shown in Figure 4 to illustrate the picture of an entire FFT algorithm. There are $\log_4 16=2$ stages and each stage has 16/4=4 butterflies in the figure.



Figure 4. An example of a 16-point Radix-4 DIF FFT



In Figure 4, the inputs are normally ordered while the outputs are presented in a digit-reversed order. In the case of Radix-4, the digits are 0, 1, 2, 3 (quaternary system). A direct result of digit-reversed order for 16-point Radix-4 DIF FFT is summarized in Table 1. For example, the output X(9) occupies the position "6" as $6_{10} = 12_4$ and the digit reversed number is $21_4 = 9_{10}$, where the subscripts denote the bases used to present the number. That is, if the input index n is written as a base-4 number, the output index is reversed base-4 number. In order to store the output data to the correct order, the position of output data needs to be replaced with the digit-reversed order. An efficient method to calculate the output indices will be introduced in Section 4 based on the bit-reversed addressing mode provided by the SC3850 core.

Index	Digital pattern	Digital reversed pattern	Digital reversed index
Index	Digital pattern	Digital revelsed pattern	
0	00	00	0
1	01	10	4
2	02	20	8
3	03	30	12
4	10	01	1
5	11	11	5
6	12	21	9
7	13	31	13
8	20	02	2
9	21	12	6
10	22	22	10
11	23	32	14
12	30	03	3
13	31	13	7
14	32	23	11
15	33	33	15

Table 1	Digital	Reversed	Order of	a 16-	point Radi	x-4 FFT
	Digitai	110101000		u 10		



Radix-4 FFT Algorithm

2.3 Radix-4 DIT FFT

It is assumed that N is a power of 4, that is, $N=4^p$. The Radix-4 DIT FFT can be derived as shown in Equation 16.

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}$$

$$\frac{N}{4} - 1 \qquad \frac{N}{4} - 1 \qquad \frac{N}{4} - 1$$

$$= \sum_{n=0} x(4n) W_N^{4nk} + \sum_{n=0} x(4n+1) W_N^{(4n+1)k} + \sum_{n=0} x(4n+2) W_N^{(4n+2)k} + \sum_{n=0} x(4n+3) W_N^{(4n+3)k}$$

$$= \sum_{n=0}^{N-1} x(4n) W_N^{4nk} + W_N^k \sum_{n=0} x(4n+1) W_N^{4nk} + W_N^{2k} \sum_{n=0} x(4n+2) W_N^{4nk} + W_N^{3k} \sum_{n=0} x(4n+3) W_N^{4nk}$$

$$= \sum_{n=0} x(4n) W_{N'4}^{nk} + W_N^k \sum_{n=0} x(4n+1) W_{N'4}^{nk} + W_N^{2k} \sum_{n=0} x(4n+2) W_{N'4}^{nk} + W_N^{3k} \sum_{n=0} x(4n+3) W_N^{4nk}$$

$$= \sum_{n=0} x(4n) W_{N'4}^{nk} + W_N^k \sum_{n=0} x(4n+1) W_{N'4}^{nk} + W_N^{2k} \sum_{n=0} x(4n+2) W_{N'4}^{nk} + W_N^{3k} \sum_{n=0} x(4n+3) W_{N'4}^{nk}$$

$$= \sum_{n=0} x(4n) W_{N'4}^{nk} + W_N^k \sum_{n=0} x(4n+1) W_{N'4}^{nk} + W_N^{2k} \sum_{n=0} x(4n+2) W_{N'4}^{nk} + W_N^{3k} \sum_{n=0} x(4n+3) W_{N'4}^{nk}$$

$$= P(k) + W_N^k Q(k) + W_N^{2k} R(k) + W_N^{3k} S(k)$$

$$k = 0, 1, 2, 3, ..., N - 1$$

Each of the sums, P(k), Q(k), R(k), and S(k), in Equation 16 is recognized as an N/4-point DFT. Although the index k ranges over N values, k=0,1,2,...,N-1, each of the sums can be computed only for k=0,1,2,...,N/4-1, since they are periodic with period N/4. The transform X(k) can be broken into four parts as shown in Equation 17.

$$\begin{split} X(k) &= P(k) + W_N^k Q(k) + W_N^{2k} R(k) + W_N^{3k} S(k) \\ X\left(k + \frac{N}{4}\right) &= P(k) + W_N^{\left(k + \frac{N}{4}\right)} Q(k) + W_N^{2\left(k + \frac{N}{4}\right)} R(k) + W_N^{3\left(k + \frac{N}{4}\right)} S(k) \\ &= P(k) - j W_N^k Q(k) - W_N^{2k} R(k) + j W_N^{3k} S(k) \\ X\left(k + \frac{2N}{4}\right) &= P(k) + W_N^{\left(k + \frac{2N}{4}\right)} Q(k) + W_N^{2\left(k + \frac{2N}{4}\right)} R(k) + W_N^{3\left(k + \frac{2N}{4}\right)} S(k) \\ &= P(k) - W_N^k Q(k) + W_N^{2k} R(k) - W_N^{3k} S(k) \\ X\left(k + \frac{3N}{4}\right) &= P(k) + W_N^{\left(k + \frac{3N}{4}\right)} Q(k) + W_N^{2\left(k + \frac{3N}{4}\right)} R(k) + W_N^{3\left(k + \frac{3N}{4}\right)} S(k) \\ &= P(k) + j W_N^k Q(k) - W_N^{2k} R(k) - j W_N^{3k} S(k) \\ k &= 0, 1, 2, ..., \frac{N}{4} - 1 \end{split}$$



Figure 5 shows the corresponding Radix-4 DIT butterfly diagram, and Figure 6 shows its simplified version.





Figure 6. Simplified Radix-4 DIT Butterfly

We can see that Figure 2 and Figure 5 are similar. Note that the principal difference between DIT and DIF butterflies is that the order of calculation has changed. In the DIF algorithm, the time domain data was "twiddled" before the sub-transforms were performed. In DIT, however, the sub-transforms are performed first, and the output is obtained by "twiddling" the resulting frequency domain data.



Radix-4 FFT Algorithm

The real and imaginary part of output data for Radix-4 DIT butterfly is specified in Equation 18.

$$\begin{aligned} Ar' &= Ar + (Cr \times Wcr - Ci \times Wci) + (Br \times Wbr - Bi \times Wbi) + (Dr \times Wdr - Di \times Wdi) \\ Ai' &= Ai + (Cr \times Wci + Ci \times Wcr) + (Br \times Wbi + Bi \times Wbr) + (Dr \times Wdi + Di \times Wdr) \\ Br' &= Ar - (Cr \times Wcr - Ci \times Wci) + (Br \times Wbi + Bi \times Wbr) - (Dr \times Wdi + Di \times Wdr) \\ Bi' &= Ai - (Cr \times Wci + Ci \times Wcr) - (Br \times Wbr - Bi \times Wbi) + (Dr \times Wdr - Di \times Wdi) \\ Cr' &= Ar + (Cr \times Wcr - Ci \times Wci) - (Br \times Wbr - Bi \times Wbi) - (Dr \times Wdr - Di \times Wdi) \\ Ci' &= Ai + (Cr \times Wci + Ci \times Wcr) - (Br \times Wbi + Bi \times Wbr) - (Dr \times Wdi + Di \times Wdr) \\ Dr' &= Ar - (Cr \times Wci + Ci \times Wcr) - (Br \times Wbi + Bi \times Wbr) + (Dr \times Wdi + Di \times Wdr) \\ Di' &= Ai - (Cr \times Wci - Ci \times Wci) - (Br \times Wbi + Bi \times Wbr) + (Dr \times Wdi + Di \times Wdr) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Br \times Wbi - Bi \times Wbi) - (Dr \times Wdr - Di \times Wdi) \end{aligned}$$

The number of multiplications and additions required by the DIT butterfly is the same as it required by the DIF butterfly, which can be seen from Figure 2 and Figure 5. Therefore, Radix-4 DIT butterfly calculation also reduces the number of complex multiplication that are needed for a N-point DFT from N² to $3(N/4)\log_4 N$ (from $4N^2$ to $3N\log_4 N$ in terms of real multiplications). In the SC3850 core, dual MACs can be used to implement the DIT algorithm efficiently. Thus, this application note will describe the implementation of the DIT algorithm on the SC3850 core in Section 4.



Figure 7. An example of a 16-point Radix-4 DIT FFT



An example of 16-point Radix-4 DIT FFT diagram is shown in Figure 7 to illustrate an entire DIT algorithm. There are $\log_4 16=2$ stages and each stage has 16/4=4 butterflies. In Figure 7, the inputs are digit-reversed ordered (refer to Table 1) while the outputs are presented in a normal order. Thus, bit-reversed addressing mode is used to load the input data samples to improve the performance of the DIT FFT algorithm.

3 SC3850 Data Types and Instructions

This section discusses the data types, SIMD instructions, and complex arithmetic in the SC3850 architecture, which can be used to efficiently implement the Radix-4 DIT FFT algorithm.

3.1 StarCore Data Types

Data Type	Size	Layout in 40-bit Register	Context	Moves
Integer	Byte (8-bit)	39 32 16 0 zero/sign 8 bits	Integer instructions, for example, IADD, ISUB, IMPY. Unaffected by saturation modes.	MOVE.B, MOVEU.B
	Word (16-bit)	39 32 16 0 zero/sign 16 bits	Integer instructions, for example, IADD, ISUB, IMPY. Unaffected by saturation modes.	MOVE.W, MOVEU.W
	Long (32-bit)	39 32 16 0 Z/S 32 bits	Integer instructions, for example, IADD, ISUB. However, multipliers are 16-bit therefore multiplication of this type is handled differently. Unaffected by saturation modes.	MOVE.L, MOVEU.L
	40-bit	39 32 16 0 40 bits	Result of an accumulation over the 32-bit boundary. Not possible to store 40-bit word as one in memory but is possible to save context of 40-bit register. 40-bit value must be adjusted before save to memory. Unaffected by saturation modes.	N/A

Table 2. Data Types Supported by the SC3850 Core



Data Type	Size	Layout in 40-bit Register	Context	Moves
Fractional	Byte (8-bit)	39 32 16 0 sign 8 bits	Fractional instructions for example, ADD, SUB, MPY, MAC. Affected by saturation mode (32/40)	MOVER.BF
	Word (16-bit)	39 32 16 0 sign 16 bits	Fractional instructions for example, ADD, SUB, MPY, MAC. Affected by saturation mode (32/40)	MOVE.F + scaling ones
	Long (32-bit)	39 32 16 0 sign 32 bits	Fractional instructions for example, ADD, SUB. However, multipliers are 16-bit therefore multiplication of this type is handled differently. Affected by saturation mode (32/40)	MOVE.L
	40-bit	39 32 16 0 40 bits	Result of an accumulation over the 32-bit boundary. Not possible to store 40-bit word as one in memory but is possible to save context of 40-bit register. 40-bit value has to be scaled, rounded etc. on or before save to memory. 40-bit value only possible with saturation mode set to 40-bit otherwise value will saturate at 32-bits.	N/A
Packed	Fractional 16-bit	39 32 16 0 S S	SIMD instructions only. 8-bit and 16-bit values can be loaded/stored. Applicable only with saturation mode 2 set to 16-bit.	MOVE2.2F
	Fractional 20-bit	39 32 16 0	SIMD instructions only. 8-bit and 16-bit values can be loaded/stored. Saturation mode 2 set to 20-bit for wide accumulation.	MOVE2.2F

Table 2. Data Types Supported by the SC3850 Core (continued)

3.2 SIMD

The SC3850 core has support for packed, or SIMD operations. The use of these instructions can improve performance since multiple data elements can be processed simultaneously.



3.2.1 SIMD Instruction Data Types

The SIMD support is for two data elements of up to 20 bits per 40-bit register as shown in Figure 8. Both byte (8-bit) and word (16-bit) data can be loaded into the 20-bit fields. The extra bits allow for increased precision.





3.2.2 SIMD Instructions

Figure 9 shows an example SIMD instruction, MAC2, which performs two separate multiplications of 16-bit sources accumulated into two 20-bit components (WH and WL) of a register.



Figure 9. Example SC3850 SIMD Instruction, MAC2



Table 3 lists the SIMD instructions in the SC3850 core.

Instruction Description		Bitwise compatible with legacy architectures
ADD2	Packed addition	yes
SUB2	Packed subtraction	yes
NEG2	Two Words Negate	yes
IMACSU2	Two integer multiply accumulate signed by unsigned	yes
PACK.2W	Packs two words	yes
PACK.2F	Packs two fractional words	yes
ADD.W	Add 16-bit or 20-bit value	no
ABS2	Two Words Absolute Value	no
ASL2	Arithmetic Shift Left by One of Two Word Operands	no
ASLL2	Multiple-Bit Arithmetic Shift Left of Two Word Operands	no
ASRR2	Multiple-Bit Arithmetic Shift Right of Two Word Operands	no
LSLL2	Multiple-Bit Bitwise Shift Left of Two Word Operands	no
LSR2	Bitwise Shift Right One Bit of Two Word Operands	no
LSRR2	Multiple-Bit Bitwise Shift Right of Two Word Operands	no
SOD2ffcc	Sum Or Difference of Two 16-Bit Values, function & cross	no
MIN2	Transfer two 16-bit minimum signed values	no
MAX2	Transfer two 16-bit maximum signed values	no
SUB.W	Subtract 16-bit or 20-bit value	no
MPY2	Multiply 2 pairs of 16-bit data.	N/A
MPY2R	Multiply 2 pairs of 16-bit data and round the lower 16 bits of the result.	N/A
MAC2	Multiply 2 pairs of 16-bit data, clip the lower 16 bits of each result into 16-bit word and accumulate it with 20-bit accumulator input	N/A
MAC2R	Multiply 2 pairs of 16-bit data, round the lower 16 bits of each result into 16-bit word and accumulate it with 20-bit accumulator input.	N/A
CLIP20	Clip two 20-bit operands.	N/A
SATU20.B	Saturate two unsigned bytes.	N/A
MAC2ffggR	Multiply 2 pairs of 16-bit data, add or subtract them from each portion	N/A
MAC2ffggl	-specific format used for FFT calculation.	N/A

Table 3. SIMD Instructions in the SC3850 Core

3.3 Complex Arithmetic

Complex arithmetic is widely used in signal processing algorithms. Figure 10 shows a convention to represent a complex number using dual 20-bit packed format. For a complex addition, two additions are needed:

```
; Addition of a+jb and c+jd to form e+jf
e=a+c;
f=b+d;
```

For a complex multiplication, two signed addition operations and four multiplications are needed.

```
; Multiplication of a+jb and c+jd to form e+jf
e=ac-bd;
f=j(bc+ad);
```







Table 4 lists instructions used for complex arithmetic:

Syntax	Description
SOD2FFCC Da,Db,Dn	Sum or Difference of Two Word Values—Function and Cross
	Performs two separate 16-bit additions or subtractions between the high and low portions
	data register. The value of FE and CC determine the behavior
	FF: A for addition and S for subtraction
	CC: XX for crossed and II for not crossed
	This instruction enables the use of the adder for smaller precision values and therefore increases the number of operations that can be performed simultaneously.
MPYRE	Assuming the complex type is stored in the register as 16-bit real, 16-bit imaginary, this computes the real part of the complex multiplication. (Da.H * Db.H) - (Da.L * Db.L) -> Dn
МРҮІМ	Assuming the complex type is stored in the register as 16-bit real, 16-bit imaginary, this computes the imaginary part of the complex multiplication. (Da.L * Db.H) + (Da.H * Db.L) -> Dn
MPYCIM	Assuming the complex type is stored in the register as 16-bit real, 16-bit imaginary, this computes the conjugate imaginary part of the complex multiplication. (Da.L * Db.H) - (Da.H * Db.L) -> Dn
MACRE	Performs MPYRE with accumulation
MACIM	Performs MPYIM with accumulation
MACCIM	Performs MPYCIM with accumulation



SC3850 Data Types and Instructions

3.4 Scaling, Rounding and Limiting

When moving fractional data to memory during DSP kernels, usually it is require to scale, round, and limit (saturate) the data on its way to the memory in order not to lose precision due to truncation that would otherwise occur. The data in the register itself is not modified to prevent accuracy loss when this register is used for subsequent operations in the core. For non-packed data types, these operations are performed by the MOVER instructions, which work in the following way:

- Scaling is done according to the scaling mode (SCM in SR). In 20-bit packed format, the scaling down uses the guard bits from the appropriate extension portion. Scaling is not performed when in the arithmetic saturation mode (SM for MOVER). Four scaling modes are supported:
 - No scaling
 - Scale down (1-bit arithmetic right shift)
 - Scale up (1-bit arithmetic left shift)
 - Scale down (2-bit arithmetic right shift)
- The data is rounded to the appropriate length (depending on the width of the transfer). Rounding is not done for 32-bit values (MOVER.L and MOEVR.2L instructions). Twos complement rounding or convergent rounding is performed according to the rounding mode bit in the SR (SR.RM).
- Limiting is done on the scaled data, looking at the value of all the MS bits that are shifted out (including guard bits for 40-bit input).

Instruction	Description
MOVER.L	Move a fractional long from a register to memory with scaling, and saturation. Note: no rounding takes place with this instruction.
MOVER.2L	Move two fractional longs from a register pair to memory with calling and saturation. Note: no rounding takes place with this instruction.
MOVER.F	Move a fractional word from a register to memory with rounding, scaling, and saturation
MOVER.2F	Move two fractional words from a register pair to memory with rounding, scaling, and saturation
MOVER.4F	Move four fractional words from a register quad to memory with rounding, scaling, and saturation
MOVERH.4F	Move four fractional words from the wide high portions of a register quad with rounding, scaling, and saturation.
MOVERL.4F	Move four fractional words from the wide low portions of a register quad with rounding, scaling and saturation.
MOVER.BF	Move a fractional byte from a register to memory with rounding, scaling, and saturation
MOVER.2BF	Move two fractional bytes from a register pair to memory with rounding, scaling, and saturation
MOVER.4BF	Move four fractional bytes from a register quad to memory with rounding, scaling, and saturation

Table 5. MOVER Instructions



4 Implementation on the SC3850 Core

In this section, an implementation of the Radix-4 DIT FFT algorithm is presented.

4.1 Scaling

The SC3850 core is a fixed-point digital signal processor. The data needs to be handled within the fixed-point range [-1, 1) in order to avoid overflow. For a Radix-4 FFT, the magnitude values computed in a butterfly stage can have a growth to $4\sqrt{2} \approx 5.657$. The real and imaginary parts of the butterfly can have a growth to 4. This is the output dynamic range. The fixed-scaling method scales down by a fixed factor at each stage to handle the bit growth. If scaling is insufficient, a butterfly output may grow beyond the dynamic range and causes an overflow. In the computation of the Radix-4 FFT on the SC3850 core, it is necessary to scale down the intermediate results by a factor of 4 to avoid any overflowing, which means that each stage of the FFT is divided by 4. If an FFT consists of M stages, the output is scaled down by 4^{M} (M = log₄(N)), where N is the length of the FFT. The scaling results in the final output are modified by the factor of $1/4^{M}$. The output sequence X'(K) (k = 0, 1, 2, 3, ..., N – 1) computed by the SC3850 processor is defined in Equation 19.

$$\begin{aligned} X'(k) &= \frac{1}{4^{M}} \sum_{n=0}^{N-1} x(n) exp\left(-j\frac{2\pi nk}{N}\right) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) exp\left(-j\frac{2\pi nk}{N}\right) \\ M &= \log_{4} N \\ k &= 0, 1, 2, 3, ..., N-1 \end{aligned}$$

For example, the total scaling amount for 1024-point DIF FFT is $1/4^{M} = 1/4^{5} = 1/1024$ (M = $\log_{4}(1024)$ = 5). Note that if a Radix-4 algorithm uses a scaling of a factor of 4, the total scaling amount is equal to the factor of 1/N. Note that the MOVER instructions introduced in Section 3.4 can be used to efficiently scale the data.

4.2 Bit-Reversed Addressing

The StarCore DSP has a bit-reversed (or reverse-carry) addressing mode to support N-point FFT addressing, where $N = 2^k$. This mode is useful for unscrambling N-point FFT data. Figure 11shows an example of reverse-carry addressing of 1024-point FFT. Before starting the reverse-carry addressing, the following registers need to be set:

1. Set "1" to the corresponding field in MCTL (Modifier Control) register

NP

Implementation on the SC3850 Core

2. Set the offset register, Nn = N / 2, where N is the number of FFT points



Figure 11. Bit-Reversed Addressing for 1024-Point FFT

The bit-reversed addressing mode is equivalent to the following process (as shown in Figure 11):

- 1. Shift the offset value in Nn to the left according to the access width. For example, if the access width is long, 2-bit left shift is performed.
- 2. Add the address register (Rn) and the offset register (Nn). Note that the carry bit is propagated from each pair of added bits in the reverse direction.



The range of values for offset register, Nn, is from 0 to $(2^{32} - 1)$, which allows reverse-carry addressing for FFTs up to 4,294,967,296 points. The base address (start address) should be aligned to N × W, where W is the number of bytes in a data element. For instance, in a 1024-point FFT on a 16-bit complex array, the base address of the array needs to be aligned to:

1,024-point \times 2-byte \times 2IQ = 4,096 bytes

Table 6 lists the 16-point FFT bit-reversed order supported by StarCore SC3850 core.

Index	Bit Pattern	Bit Reversed Pattern	Bit Reversed Index
0	0000	0000	0
1	0001	1000	8
2	0010	0100	4
3	0011	1100	12
4	0100	0010	2
5	0101	1010	10
6	0110	0110	6
7	0111	1110	14
8	1000	0001	1
9	1001	1001	9
10	1010	0101	5
11	1011	1101	13
12	1100	0011	3
13	1101	1011	11
14	1110	0111	7
15	1111	1111	15

Table 6.	Bit-Reversed	Order	Supported by	StarCore	SC3850	DSPs
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As mentioned in Section 2.3, the data are digit-reversed ordered in the Radix-4 DIT algorithm. A comparison between the digit-reversed order for Radix-4 (Table 1) and the bit-reversed order supported by the SC3850 DSP (Table 6) shows that the two middle output indices are interchanged. This means that the bit-reversed addressing mode can be used for the digit-reversed ordering in Radix-4 FFT with the exchange of the two middle indices. For this purpose, the algorithm interchanges the order of B and C of Radix-4 DIT butterfly as shown in Figure 12. Otherwise, the usual digital-reversed addressing is used.



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Figure 12. Radix-4 DIT Butterflies: (A) Normal Ordered Input and (B) Reordered Data for Digit-Reverse Addressing

To further improve the process of loading input data, two address registers in bit-reversed mode can be used to load the input data in parallel. The offset between the two address registers is equal to N/2 x complex data width (4 bytes). To update the registers and get next input data, +Ni/2 is used for both offset registers instead of +Ni.

An example to illustrate how to access 16 input data with two bit-reversed address registers is given in Table 7. For comparison, digital-reversed index and bit-reversed index with one address register are also listed in the table. In the fourth column of the table, r0 and r1 are used in bit-reversed addressing mode. Register r0 is used to access index 0, 4, 2, 6, 1, 5, 3, 7 and r1 is used to access index 8, 12, 10, 14, 9, 13, 11, 15. Note that the offset between r0 and r1 is 8, which is half of N (N=16 in this example).

Index	Digital-Reversed Index	Bit-Reversed Index with One Address Register	Bit-Reversed Index with Two Address Registers R0, R1
0	0	0	0 (r0)
1	4	8	4 (r0)
2	8	4	8 (r1)
3	12	12	12 (r1)

Table	7.	Two	Registers	in	Bit-Reversed	Addressing	Mode
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Index	Digital-Reversed Index	Bit-Reversed Index with One Address Register	Bit-Reversed Index with Two Address Registers R0, R1
4	1	2	2 (r0)
5	5	10	6 (r0)
6	9	6	10 (r1)
7	13	14	14 (r1)
8	2	1	1 (r0)
9	6	9	5 (r0)
10	10	5	9 (r1)
11	14	13	13 (r1)
12	3	3	3 (r0)
13	7	11	7 (r0)
14	11	7	11 (r1)
15	15	15	15 (r1)

Table 7. Two Registers in Bit-Reversed Addressing Mode (continued)

The input data is a vector of N complex values represented as 16-bit two's complement numbers that are decomposed as the real and imaginary components of the data sample. The implemented Radix-4 DIT butterfly equations are summarized in Equation 20.

$$\begin{aligned} Ar' &= Ar + (Cr \times Wcr - Ci \times Wci) + (Br \times Wbr - Bi \times Wbi) + (Dr \times Wdr - Di \times Wdi) \\ Ai' &= Ai + (Cr \times Wci + Ci \times Wcr) + (Br \times Wbi + Bi \times Wbr) + (Dr \times Wdi + Di \times Wdr) \\ Br' &= Ar + (Cr \times Wcr - Ci \times Wci) - (Br \times Wbr - Bi \times Wbi) - (Dr \times Wdr - Di \times Wdi) \\ Bi' &= Ai + (Cr \times Wci + Ci \times Wcr) - (Br \times Wbi + Bi \times Wbr) - (Dr \times Wdi + Di \times Wdr) \\ Cr' &= Ar - (Cr \times Wcr - Ci \times Wci) + (Br \times Wbi + Bi \times Wbr) - (Dr \times Wdi + Di \times Wdr) \\ Ci' &= Ai - (Cr \times Wci + Ci \times Wcr) - (Br \times Wbi - Bi \times Wbi) + (Dr \times Wdr - Di \times Wdi) \\ Dr' &= Ar - (Cr \times Wcr - Ci \times Wci) - (Br \times Wbi + Bi \times Wbr) + (Dr \times Wdr - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wcr - Ci \times Wci) - (Br \times Wbi + Bi \times Wbr) + (Dr \times Wdi + Di \times Wdr) \\ Di' &= Ai - (Cr \times Wcr - Ci \times Wci) + (Br \times Wbi - Bi \times Wbi) + (Dr \times Wdi - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wcr - Ci \times Wci) + (Br \times Wbi - Bi \times Wbr) + (Dr \times Wdi - Di \times Wdi) \\ Dr' &= Ar - (Cr \times Wcr - Ci \times Wci) + (Br \times Wbi - Bi \times Wbr) + (Dr \times Wdi - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Br \times Wbi - Bi \times Wbi) - (Dr \times Wdi - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Br \times Wbi - Bi \times Wbi) - (Dr \times Wdi - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Br \times Wbi - Bi \times Wbi) - (Dr \times Wdr - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Br \times Wbr - Bi \times Wbi) - (Dr \times Wdr - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Br \times Wbr - Bi \times Wbi) - (Dr \times Wdr - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Br \times Wbr - Bi \times Wbi) - (Dr \times Wdr - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Br \times Wbr - Bi \times Wbi) - (Dr \times Wdr - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Br \times Wbr - Bi \times Wbi) - (Dr \times Wdr - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Br \times Wbr - Bi \times Wbi) - (Dr \times Wdr - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Br \times Wbr - Bi \times Wbi) - (Dr \times Wdr - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Dr \times Wbi + Bi \times Wbi) + (Dr \times Wdr - Di \times Wdi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Dr \times Wbi + Bi \times Wbi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Dr \times Wbi + Bi \times Wbi) \\ Di' &= Ai - (Cr \times Wci + Ci \times Wcr) + (Dr \times Wbi + Bi \times Wbi) \\ Di' &= Ai - (C$$

Equation 20 shows the implementation of Radix-4 DIT butterfly equations on the SC3850 core. The output order of the implementation is different from the original order in Equation 18. Please note the interchanging locations of B and C enables the use of StarCore supported bit-reversed addressing. This means that instead of storing the output of Radix-4 DIT butterfly in the order of A', B', C', D', they are stored in the order of A', C', B', D', as shown in Figure 12.



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4.3 Implementation of Radix-4 DIT FFTs

Figure 13 shows the actual process of calculation for 1024-point FFT as an example.



Figure 13. Diagram of 1024-point FFT Calculation Using Radix-4 DIT Algorithm

In this implementation, the FFT is summarized as follows:

- 1. First stage: The input data are loaded in the bit-reversed addressing mode. The Radix-4 DIT butterfly calculation is repeated for N/4 times on the input data. In the first stage, the twiddle factors are all equal to 1. As a result, there is no multiplication in the first stage. The output can be scaled down by 0, 2, and 4 depending on the input parameter of the FFT function.
- 2. Middle stage: The N/4 times Radix-4 DIT butterfly calculation is repeated for $(\log_4 N 2)$ times. The twiddle factors vary with stages. In the second stage, the twiddle factors are equal to $(W_{16}^{k}, W_{16}^{2k}, W_{16}^{3k})$ (k = 0, 1, 2, 3). In the third stage, the twiddle factors are $(W_{64}^{k}, W_{64}^{2k}, W_{64}^{3k})$ (k = 0, 1, 2, ..., 15). In the fourth stage, the twiddle factors are $(W_{256}^{k}, W_{256}^{2k}, W_{256}^{3k})$ (k = 0, 1, 2, ..., 63), and so on. The output at each stage can be scaled down by 0, 2, and 4 depending on the input parameter of the FFT function.
- 3. Last stage: The Radix-4 DIT butterfly calculation is repeated for N/4 times. The twiddle factors are $(W_N^{\ k}, W_N^{\ 2k}, W_N^{\ 3k})$ (k = 0, 1, 2, ..., N/4-1) in this stage. The output can be scaled down by 0, 2, and 4 depending on the input parameter of the FFT function.

The routine uses one first stage, $\log_4 N-2$ middle stages, and one last stage to perform the radix-4 FFT algorithm, as shown in Figure 13. As we can see from the figure that there are N/4 butterflies in each stage. The N/4 butterflies can be classified into groups depending on which stage they are at. In the first stage, all twiddle factors are equal to one. Thus one loop is used to compute N/4 butterflies without loading the twiddle factors. In the middle stages, the butterflies with the same twiddle factors are grouped together to reduce the memory access. The radix-4 butterflies are classified into 4 groups for the first middle stage, 16 groups for the second middle stage, and so on. Therefore, the middle stages are written using three loops. The outermost loop "k" cycles through the (log₄N-2) middle stages. The loop "j" cycles through the groups



of butterflies with different twiddle factors, and loop "i" reuses the twiddle factors for the different butterflies within a stage. In the last stage, only one butterfly exist in each group, and thus one loop is used to go through all the groups. Table 8 shows the grouped butterflies at different stages.

Stage	Groups with Different Twiddle Factors	Butterflies with Common Twiddle Factors	Groups * Butterflies
First Stage	1	N/4	N/4
Middle Stage 1	4	N/16	N/4
Middle Stage 2	16	N/64	N/4
Middle Stage log ₄ N-2	N/16	4	N/4
Last Stage	N/4	1	N/4

Table 8.	Grouped	Butterflies	at	Different	Stages
----------	---------	--------------------	----	-----------	--------

Input and output data are 16-bit complex array, each of length N. Input and output data buffers are cyclic for 2*N complex inputs with the modulo addressing mode, as shown in Figure 14. Each stage uses the input and output data buffer one after the other. Therefore, when the stage, #n, uses the input data buffer as the input point and the output data buffer as the output point, the next stage, #n + 1, uses the output data buffer as the input data buffer as the output point. The final output data are stored in input data buffer for $\log_4(N) =$ even or in output data buffer for $\log_4(N) =$ odd.

FFT Points

256

1024

Final Output Data

Input Data Buffer

Output Data Buffer



Cyclic Butter	Cyc	lic	В	uff	е	ľ
---------------	-----	-----	---	-----	---	---

Figure	14 C	velie	Ruffor	for	Innut	and	Output	Data
rigule	14. 0	yunu	Duner	101	mpuι	anu	ουιραι	ναια



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4.4 Optimization Used in FFT Implementation

The optimization techniques improve the performance by taking special advantage of StarCore parallel computation capability. The SC3850 cores efficiently deploy a Variable Length Execution Set (VLES) instruction set. The SC3850 architecture has:

- Four parallel ALUs, each is capable of performing dual MAC and most other arithmetic operations
- Two Address Arithmetic Units (AAUs) for address arithmetic operations

A VLES can contain up to four data ALU instructions and two AAU instructions. To maximize the computational performance, four ALUs and two AAUs should be utilized simultaneously as much as possible. As the StarCore architecture has high instruction-level parallelism, it is possible to schedule independent blocks in parallel to increase performance.

Code optimization also considers the memory structure to improve the performance. The SC3850 architecture provides a total sixteen 40-bit data registers, D0-D15 and sixteen 32-bit address registers, R0-R15. The dual Harvard architecture in the SC3850 core is capable to access up to two 64-bit data per cycle.

The following subsections present the optimization techniques which were used to increase the speed of Radix-4 DIT FFT algorithm.

4.4.1 Basic Implementation of Radix-4 DIT Butterfly: SIMD Instructions and Parallel Computing

The multiplication and addition operations in the Radix-4 DIT butterfly can be calculated in parallel using the multiple ALUs in the SC3850 DSP. The SC3850 DSP has some instructions which enable faster implementation of the Radix-4 DIT butterfly algorithms. The instructions used for the computation are shown in Table 9.

Instruction	Description
SOD2FFCC	Sum Or Difference of Two 16-Bit Values, function & cross
MAC2ffggR	SIMD2 Signed Fractional Multiply and Wide Accumulate - Real
MAC2ffggl	SIMD2 Signed Fractional Multiply and Wide Accumulate - Imaginary
MOVE2.2F	Transfers 2 16-bit fractional data between the memory and one data registers in packed 20-bit format, in a single 32-bit access
MOVE2.4F	Transfers 4 16-bit fractional data between the memory and two data registers in packed 20-bit format, in a single 64-bit access
MOVERH.4F	Move Four Wide High Fractional Words to Memory With Scaling, Rounding, and Saturation (AGU)
MOVERL.4F	Move Four Wide Low Fractional Words to Memory With Scaling, Rounding, and Saturation (AGU)

Table 9.	SC3850	SIMD	Instructions	for	FFT
		•			

The basic implementation on the SC3850 DSP is shown in Example 1. The **move2.4f** and **move2.2f** instructions are used to load the real and imaginary parts of input samples. The **moverh.4f** and **moverl.4f** instructions are used to store the real and imaginary parts of output samples. Single Instruction Multiple



Data (SIMD) instructions, **mac2ffggr** and **mac2ffggi**, can be used for two 16-bit multiplication or accumulation between the high and low portions of two sources (real and imaginary).

```
Example 1. Radix-4 DIT Butterfly Implementation
```

```
; Radix-4 DIT FFT (SIMD Instruction and Parallel Computing
Γ
   move2.2f (r1),d2
                              ; Load WC1
   move2.4F (r0)+,d0:d1
                             ; Load IA1:IC1
   ]
   [
   move2.4f (r0)+,d8:d9
                              ; Load IB1:ID1
   move2.4f (r2)+n2,d10:d11
                              ; Load WB1:WD1
   1
   Γ
                              ; M1_1.H = IA[re] + IC[re] * WC[re] - IC[im] * WC[im]
   MAC2ASSAR d1,d2,d0.H,d12
                              ; M1 1.L = IA[re] - IC[re] * WC[re] + IC[im] * WC[im]
   MAC2AASSI d1,d2,d0.L,d13
                              ; M1_2.H = IA[im] + IC[re] * WC[im] + IC[im] * WC[re]
                               ; M1_2.L = IA[im] - IC[re] * WC[im] - IC[im] * WC[re]
   ]
   Γ
   MAC2ASSAR d8,d10,d12.H,d12 ; M2 1.H = M1 1.H + IB[re] * WB[re] - IB[im] * WB[im]
                              ; M2 1.L = M1 1.H - IB[re] * WB[re] + IB[im] * WB[im]
   MAC2AASSI d8,d10,d12.L,d4
                              ; M2_2.H = M1_1.L + IB[re] * WB[im] + IB[im] * WB[re]
                              ; M2_2.L = M1_1.L - IB[re] * WB[im] - IB[im] * WB[re]
   MAC2AASSI d8,d10,d13.H,d13 ; M2_3.H = M1_2.H + IB[re] * WB[im] + IB[im] * WB[re]
                              ; M2_3.L = M1_2.H - IB[re] * WB[im] - IB[im] * WB[re]
   MAC2SAASR d8,d10,d13.L,d5
                              ; M2_4.H = M1_2.L - IB[re] * WB[re] + IB[im] * WB[im]
                               ; M2_4.L = M1_2.L + IB[re] * WB[re] - IB[im] * WB[im]
   1
   Γ
   MAC2ASSAR d9,d11,d12
                              ; OA[re] = M2 1.H + ID[re] * WD[re] - ID[im] * WD[im]
                              ; OB[re] = M2 1.L - ID[re] * WD[re] + ID[im] * WD[im]
                              ; OC[re] = M2_2.H - ID[re] * WD[im] - ID[im] * WD[re]
   MAC2SSAAI d9,d11,d4
                              ; OD[re] = M2_2.L + ID[re] * WD[im] + ID[im] * WD[re]
                              ; OA[im] = M2 3.H + ID[re] * WD[im] + ID[im] * WD[re]
   MAC2AASSI d9.d11.d13
                              ; OB[im] = M2_3.L - ID[re] * WD[im] - ID[im] * WD[re]
   MAC2ASSAR d9,d11,d5
                              ; OC[im] = M2 4.H + ID[re] * WD[re] - ID[im] * WD[im]
                               ; OD[im] = M2_4.L - ID[re] * WD[re] + ID[im] * WD[im]
   ]
   Γ
   MOVERH.4F d12:d13:d6:d7, (r4)+n0 ; save OA1:OA2
   MOVERL.4F d12:d13:d6:d7, (r5)+n0 ; save OB1:OB2
   1
   Γ
                                  ; save OC1:OC2
   MOVERH.4F d4:d5:d14:d15,(r4)+n1
   MOVERL.4F d4:d5:d14:d15,(r5)+n1
                                    ; save OD1:OD2
   1
```

In the implementation, the parallel computing technique is used to maximize the usage of the StarCore multiple ALUs for the calculation of independent output values that have an overlap input source data values. For the Radix-4 DIT butterfly, A', C', B', and D' can be calculated in parallel.

In the StarCore architecture, most element combinations that are supported have data width up to 64-bit. This means that four short words (4×16 -bits) or two long words (2×32 -bits) can be fetched at the same time. For example, the **move2.4f** and **moverh.4f** instructions are used in the above code to efficiently load and store the data samples. For this purpose, the memory addresses should be aligned to the access width



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of the instructions used. For example, 8-byte ($4 \times$ short words) accessing should be aligned to the 8-byte addresses.

4.4.2 Software Pipelining

Software pipelining is an optimization technique where a sequence of instructions is transformed into a pipeline of several copies of the sequence. The sequences then work in parallel to leverage more of the available parallelism of the architecture. Software pipelined Radix-4 DIT butterfly is shown in Example 2. Two set of Radix-4 DIT butterflies can be calculated inside one loop with the software pipelining.

```
Example 2. Software Pipelined Radix-4 DIT Butterfly
```

```
; Radix-4 DIT FFT (Software Pipelining)
FALIGN
start loop2 stage3:
   loopstart0
       [ ;01
       MAC2ASSAR d1,d2,d0.H,d12
                               ; M1 1.H = IA[re] + IC[re] * WC[re] - IC[im] * WC[im]
                               ; M1 1.L = IA[re] - IC[re] * WC[re] + IC[im] * WC[im]
                               ; M1_2.H = IA[im] + IC[re] * WC[im] + IC[im] * WC[re]
      MAC2AASSI d1,d2,d0.L,d13
                               ; M1 2.L = IA[im] - IC[re] * WC[im] - IC[im] * WC[re]
       MOVERH.4F d12:d13:d6:d7,(r4)+n0
                                   ; save OA1:OA2
       MOVERL.4F d12:d13:d6:d7,(r5)+n0
                                     ; save OB1:OB2
       1
       [ ;02
       MAC2ASSAR d8,d10,d12.H,d12
                                ; M2_1.H = M1_1.H + IB[re] * WB[re] - IB[im] * WB[im]
                                 ; M2_1.L = M1_1.H - IB[re] * WB[re] + IB[im] * WB[im]
       MAC2AASSI d8,d10,d12.L,d4
                                 ; M2_2.H = M1_1.L + IB[re] * WB[im] + IB[im] * WB[re]
                                 ; M2_2.L = M1_1.L - IB[re] * WB[im] - IB[im] * WB[re]
                                ; M2 3.H = M1 2.H + IB[re] * WB[im] + IB[im] * WB[re]
       MAC2AASSI d8,d10,d13.H,d13
                                 ; M2 3.L = M1 2.H - IB[re] * WB[im] - IB[im] * WB[re]
       MAC2SAASR d8,d10,d13.L,d5
                                 ; M2_4.H = M1_2.L - IB[re] * WB[re] + IB[im] * WB[im]
                                 ; M2 4.L = M1 2.L + IB[re] * WB[re] - IB[im] * WB[im]
       MOVERH.4F d4:d5:d14:d15,(r4)+n1 ; save OC1:OC2
       MOVERL.4F d4:d5:d14:d15,(r5)+n1 ; save OD1:OD2
       ]
       [;03
       MAC2ASSAR d9,d11,d12
                               ; OA[re] = M2_1.H + ID[re] * WD[re] - ID[im] * WD[im]
                               ; OB[re] = M2 1.L - ID[re] * WD[re] + ID[im] * WD[im]
                               ; OC[re] = M2 2.H - ID[re] * WD[im] - ID[im] * WD[re]
       MAC2SSAAI d9.d11.d4
                               ; OD[re] = M2 2.L + ID[re] * WD[im] + ID[im] * WD[re]
       MAC2AASSI d9,d11,d13
                               ; OA[im] = M2 3.H + ID[re] * WD[im] + ID[im] * WD[re]
                               ; OB[im] = M2_3.L - ID[re] * WD[im] - ID[im] * WD[re]
       MAC2ASSAR d9,d11,d5
                               ; OC[im] = M2_4.H + ID[re] * WD[re] - ID[im] * WD[im]
                               ; OD[im] = M2_4.L - ID[re] * WD[re] + ID[im] * WD[im]
       MOVE2.4F (r0)+,d0:d1
                               ; Load IA2:IC2
       move2.4f (r2)+,d10:d11
                               ; Load WB2:WD2
       ]
; Another Radix-4 DIT butterfly calculation
[;04
       MAC2ASSAR d1,d3,d0.H,d6 ; M1_1.H = IA[re] + IC[re] * WC[re] - IC[im] * WC[im]
                               ; M1 1.L = IA[re] - IC[re] * WC[re] + IC[im] * WC[im]
       MAC2AASSI d1,d3,d0.L,d7
                               ; M1_2.H = IA[im] + IC[re] * WC[im] + IC[im] * WC[re]
                               ; M1_2.L = IA[im] - IC[re] * WC[im] - IC[im] * WC[re]
       move2.4f (r0)+,d8:d9
                               ; Load IB2:ID2
       move2.4f (r1)+,d2:d3
                               ; Load WC1:WC2
```

```
1
    [;05
   MAC2ASSAR d8,d10,d6.H,d6
                                ; M2 1.H = M1 1.H + IB[re] * WB[re] - IB[im] * WB[im]
                                ; M2 1.L = M1 1.H - IB[re] * WB[re] + IB[im] * WB[im]
   MAC2AASSI d8,d10,d6.L,d14 ; M2_2.H = M1_1.L + IB[re] * WB[im] + IB[im] * WB[re]
                                ; M2 2.L = M1 1.L - IB[re] * WB[im] - IB[im] * WB[re]
                                ; M2 3.H = M1 2.H + IB[re] * WB[im] + IB[im] * WB[re]
   MAC2AASSI d8,d10,d7.H,d7
                                ; M2 3.L = M1 2.H - IB[re] * WB[im] - IB[im] * WB[re]
   MAC2SAASR d8,d10,d7.L,d15 ; M2_4.H = M1_2.L - IB[re] * WB[re] + IB[im] * WB[im]
                                ; M2_4.L = M1_2.L + IB[re] * WB[re] - IB[im] * WB[im]
                                ; Load IA1:IC1
   MOVE2.4F (r0)+.d0:d1
   1
    [ ;06
                                ; OA[re] = M2 1.H + ID[re] * WD[re] - ID[im] * WD[im]
   MAC2ASSAR d9,d11,d6
                                ; OB[re] = M2_1.L - ID[re] * WD[re] + ID[im] * WD[im]
                                ; OC[re] = M2 2.H - ID[re] * WD[im] - ID[im] * WD[re]
   MAC2SSAAI d9,d11,d14
                                ; OD[re] = M2 2.L + ID[re] * WD[im] + ID[im] * WD[re]
   MAC2AASSI d9,d11,d7
                                ; OA[im] = M2 3.H + ID[re] * WD[im] + ID[im] * WD[re]
                                ; OB[im] = M2 3.L - ID[re] * WD[im] - ID[im] * WD[re]
   MAC2ASSAR d9,d11,d15
                                ; OC[im] = M2_4.H + ID[re] * WD[re] - ID[im] * WD[im]
                                ; OD[im] = M2_4.L - ID[re] * WD[re] + ID[im] * WD[im]
   move2.4f (r0)+,d8:d9
                                ; Load IB1:ID1
   move2.4f (r2)+,d10:d11
                                ; Load WB1:WD1
   ]
loopend0
```

On a Change of Flow (COF), the core has only filled one fetch buffer. If the destination VLES is within two fetch sets, the core will have to fill another fetch buffer and that will cause a stall. To avoid this situation, ensure that the destination VLES of all COF operations are contained within one fetch set. The **falign** directive will pad the previous VLES with NOP to ensure the following VLES is contained within one fetch set. Please note that **falign** is used before the loop in above example code.

4.4.3 Twiddle Factors

Twiddle factors are generated with a fixed scale factor; i.e., $32768 = 2^{15}$ for the 16-bit FFT functions. The twiddle factors are pre-calculated. The arrays for the complex input data, complex output data, and twiddle factors must be double-word aligned.

As mentioned in Section 4.3, three loops are used to go through the grouped radix-4 butterflies in the middle stages. In the inner loop, the FFT algorithm accesses the three twiddle factors $(W_b, W_c, and W_d)$ per iteration, as the butterflies that reuse twiddle factors are lumped together. As a result, the number of memory accessed is reduced.



```
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```

The C code in Example 3 is used to generate the twiddle factors.

Example 3. C Code to Generate Twiddle Factors

```
#define pi 3.1415926535897931
void twiddle_factors_create()
{
         short twiddle B[N/2];
         short twiddle C[N/2];
         short twiddle_D[N/2];
         FILE *stream_twiddle_Wc_print_out;
         FILE *stream_twiddle_Wbd_print_out;
         for(i=0;i<N/4;i++)// Wc
          {
                   twiddle_C[2*i+0] = (short)min(my_round(cos(-2*pi/N*2*i)*0x0008000),0x7FFF);
                   twiddle_C[2*i+1] = (short)min(my_round(sin(-2*pi/N*2*i)*0x00008000),0x7FFF);
          }
         for(i=0;i<N/4;i++)// Wb
          {
                   twiddle_B[2*i+0] = (short)min(my_round(cos(-2*pi/N*1*i)*0x00008000),0x7FFF);
                   twiddle_B[2*i+1] = (short)min(my_round(sin(-2*pi/N*1*i)*0x0008000),0x7FFF);
         for(i=0;i<N/4;i++)// Wd
                   twiddle D[2*i+0] = (short)min(my round(cos(-2*pi/N*3*i)*0x0008000),0x7FFF);
                   twiddle_D[2*i+1] = (short)min(my_round(sin(-2*pi/N*3*i)*0x0008000),0x7FFF);
          }
\#if(N=256)
         stream twiddle Wc print out = fopen( "256/wctwiddles 256 printed.dat", "w" );
         stream twiddle Wbd print out = fopen( "256/wbdtwiddles 256 printed.dat", "w" );
#endif
         // #if (N==256)
#if (N==1024)
         stream twiddle Wc print out = fopen( "1024/wctwiddles 1024 printed.dat", "w" );
         stream twiddle Wbd print out = fopen( "1024/wbdtwiddles 1024 printed.dat", "w" );
#endif
         // #if (N==1024)
#if (N==4096)
         stream_twiddle_Wc_print_out = fopen( "4096/wctwiddles_4096_printed.dat", "w" );
         stream_twiddle_Wbd_print_out = fopen( "4096/wbdtwiddles_4096_printed.dat", "w" );
#endif
         // #if (N==4096)
         for (i=0; i < N/4; i++)
         fprintf(stream_twiddle_Wc_print_out, "%04x\n",0x0000FFFF&(int)twiddle_C[2*i+0]);
         fprintf(stream twiddle Wc print out, "%04x\n",0x0000FFFF&(int)twiddle C[2*i+1]);
         fprintf(stream twiddle Wbd print out,"%04x\n",0x0000FFFF&(int)twiddle B[2*i+0]);
         fprintf(stream_twiddle_Wbd_print_out,"%04x\n",0x0000FFFF&(int)twiddle_B[2*i+1]);
         fprintf(stream_twiddle_Wbd_print_out,"%04x\n",0x0000FFFF&(int)twiddle_D[2*i+0]);
         fprintf(stream_twiddle_Wbd_print_out,"%04x\n",0x0000FFFF&(int)twiddle_D[2*i+1]);
         fclose(stream_twiddle_Wc_print_out);
         fclose(stream twiddle Wbd print out);
         return;
```



4.5 FFT Reference Code

In the first stage, the input data A and B are loaded with address register r0, and input data C and D are loaded address registers r3. As described in Section 4.2, bit-reversed addressing mode is used for registers r0 and r3 to reorder the input data. The output data A', B', C', and D' are stored to memory using the address registers, r4, r5, r8, and r9, respectively. The diagram of the first stage is illustrated in Figure 15.



Figure 15. Diagram of the First Stage

Figure 16 shows the diagram of the middle and the last stages. Although different number of loops are used for the middle and the last loops, the same registers are used to load and store data for the DIT butterfly. Thus the middle stages and the last stage share the same diagram.







The procedure of Radix-4 DIT butterfly calculation is summarized as follows:

- 1. The input data [A, B, C, D] are loaded from memory. MOVE2.4F instruction is used to load the real and imaginary part in parallel.
- 2. In the first stage, SOD2ffcc computes the butterfly shown in Figure 15. The SOD2ffcc instruction performs two separate 16-bit additions or subtractions between the high and low portions of two source data registers, and stores the results in the two portions of the destination data register.
- 3. In the middle and last stages, MAC2ffggR and MACffggI instructions are used to compute the butterfly shown in Figure 16. The functionality of the MAC2ffggR and MACffggI are shown in Figure 17 and Figure 18, respectively.



Figure 17. MAC2AASSR Da, Db, Ds. H/L, Dn



Figure 18. MAC2AASSI Da, Db, Ds. H/L, Dn



4. MOVERH.4F and MOVERL.4F instructions are used to scale, round, and limit the output data samples and store they into memory. The scaling down factors can be 1, 2, 4.

The Radix-4 DIT FFT source code is shown Example 4. The arguments of this function are the pointer of data buffer (input and output), the pointers of twiddle factors, the number of FFT points, the number of stages, and the shift bits. As mentioned in Section 4.1, scaling can be used to avoid overflowing at each stage. This is why we have the shift as an input parameter of the FFT function. Shift can be 0, 1, and 2 for no scaling, scaling down by 2, and scaling down by 4, respectively.

Example 4. Radix-4 FFT Source Code

```
OPT BE
   SECTION .text
 -----BENCHMARK------
                                                   GLOBAL _sc3850_fft_radix4_complex_16x16_asm
   FALIGN
_sc3850_fft_radix4_complex_16x16_asm_begin:
sc3850 fft radix4 complex 16x16 asm type
                                                       ;// begin to count cycles
                                            func
benchmark:
init:
    Γ
   move.w (SP-14),d14
                                ; N
   move.l r0,d0
                              ; data_buffer
    push.2l r6:r7
;
   1
   ]
   push.21 d6:d7
                        ;// scaling OFF,
   move.l #$00e41008.sr
                                              SM=0, SM2=0, two's-complement rounding, W20-bits mode ON
    move.l #$00e41018,sr ;// scaling by1 ON, SM=0, SM2=0, two's-complement rounding, W20-bits mode ON
;
                        ;// scaling by ON, SM=0, SM2=0, two's-complement rounding, W20-bits mode ON
    move.l #$00e41038,sr
;
    move.l #$00e41088,sr
                          ;// scaling OFF,
                                             SM=0, SM2=1, two's-complement rounding, W20-bits mode ON
;
                                           SM=0, SM2=1, two's-complement rounding, W20-bits mode OFF
    move.l #$00e40088,sr
                          ;// scaling OFF,
;
   move.1 #$00e41018,sr ;// scaling ON,
                                             SM=0, SM2=0, two's-complement rounding, W20-bits mode ON
;
                                            SM=0, SM2=0, two's complement rounding, W20-bits mode ON
    move.l #$00e41098,sr ;// scaling ON,
;
   1
   Γ
   asl d14,d1
                               ; N*2
   add d14,d0,d4
                               ; ->IB1 = data buffer+N*1
   asr d14,d15
                                ; N/2
   push MCTL
   move.l #($00001001),MCTL
                               ; set r0, r3 in bit reverse
   1
   Г
   asr d15,d13
                                ; N/4
   add d14,d4,d2
                               ; ->IC1 = data buffer+N*2
   asl d1,d3
                              ; N*4
                              ; load m0=N
   move.l d14,m0
   move.w (SP-34),r2
                               ; r2 = Shift
   ]
    [
   asr d13,d12
                               ; N/8
                               ; N*8
   asl d3.d3
                               ; ->OA1 = data_buffer+N*4
   add d3,d0,d4
                               ; load m1=N*2
   move.l d1,m1
   dosetup1 _start_loop1_stage2
   1
   Γ
   add d13,d15,d13
                               ; N/2+N/4
   add d15,d4,d5
                               ; ->OD1 = data_buffer+N*4+N/2
   add d14,d4,d6
                               ; ->OC1 = data buffer+N*5
   move.l d2,r3
                               ; r3 = data_buffer+N*2
```

```
NP
```

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```
dosetup2 _start_loop2_stage2
   1
   [
   asr d12,d9
                            ; N/16
   asr d12,d11
                            ; N/16
   neg d12
                            ; -N/8
   add d14,d6,d6
                           ; ->OB1 = data buffer+N*6
   move.l d4,r4
                           ; data buffer+N*4 = r4
   move.l d12,n0
                            ; n0=N/8
   1
                           ; if (Shift==1), T=1
   cmpeqa.w #1,r2
   ift move.l #$00e41018,sr ; scaling by1 ON, SM=0, SM2=0, two's-complement rounding, W20-bits mode ON
                           ; if (Shift==2), T=1
   cmpeqa.w #2,r2
                         ; scaling by2 ON, SM=0, SM2=0, two's-complement rounding, W20-bits mode ON
   ift move.l #$00e41038,sr
   ſ
                            ; ->OB_N/2 = data_buffer+N*6+N/2
   add d15,d6,d7
                            ; 1-N/8
   add #1,d12
   addnc.w #-2,d11,d10
                           ; N/16-2
   asrr #2,d14
                            ; N/4
   push r4
                           ; data_buffer+N*4
   tfra r0,b4
                            ; data_buffer
   ]
   [
                         ; (N/16-2)*2
   asl d10,d10
                            ; ->OB1 = data_buffer+N*6
   move.l d6,r8
                            ; ->OA_N/2 = data_buffer+N*4+N/2
   move.l d5,r5
   ]
   Γ
                          ; load m2=N*8
   move.l d3,m2
   move.l d13,n3
                            ; n3=N/2+N/4,
   1
   [
   move.l d11,n2
                            ; N/16
   move.l d10,r15
                            ; (N/32-1)*4 = r15
   1
kernel:
; FIRST LOOP (RADIX 4)
   ; Wb=Wd=Wc=1
   [
   move.l d12,n1
                            ; n1=1-N/8
   MOVE2.4F (r0)+n0,d0:d1
                            ; load IA1:IA_N/2
   1
   [
   sub #1,d9
   MOVE2.4F (r3)+n0,d4:d5 ; load IC1:IC_N/2
   MOVE2.4F (r0)+n0,d2:d3
                            ; load IB1:IB_N/2
   ]
   [
   sod2aaii d4,d0,d0
                            ; IA1+IC1
                            ; IA1-IC1
   sod2ssii d4,d0,d4
                            ; IA_N/2+IC_N/2
   sod2aaii d5,d1,d1
   sod2ssii d5,d1,d5
                           ; IA_N/2-IC_N/2
   move.l d7,r9
                            ; ->OB N/2 = data buffer+N*6+N/2
   MOVE2.4F (r3)+n0,d6:d7
                           ; load ID1:ID_N/2
   1
   Γ
                           ; IB1+ID1
   sod2aaii d6,d2,d2
                           ; IB1-ID1
   sod2ssii d6,d2,d6
   sod2aaii d7,d3,d3
                            ; IB_N/2+ID_N/2
   sod2ssii d7,d3,d7
                            ; IB_N/2-ID_N/2
   doen3 d9
```



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Implementation on the SC3850 Core
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```
MOVE2.4F (r0)+n0,d8:d9
                               ; load IA2:IA N/2+1
   1
    [
   sod2aaii d2,d0,d0
                               ; IA1+IC1+(IB1+ID1) = OA1
   sod2ssii d2,d0,d2
                               ; IA1+IC1-(IB1+ID1) = OB1
                               ; IA1-IC1-j(IB1-ID1) = OC1
   sod2saxx d6,d4,d4
   sod2asxx d6,d4,d6
                              ; IA1-IC1+j(IB1-ID1) = OD1
   MOVE2.4F (r3)+n0,d12:d13
                            ; load IC2:IC N/2+1
   MOVE2.4F (r0)+n0,d10:d11 ; load IB2:IB_N/2+1
   1
   Γ
   sod2aaii d12,d8,d8
                               ; IA2+IC2
   sod2ssii d12,d8,d12
                               ; IA2-IC2
   sod2aaii d13,d9,d9
                               ; IA_N/2+1 + IC_N/2+1
   sod2ssii d13,d9,d13
                               ; IA_N/2+1 - IC_N/2+1
   MOVE2.4F (r3)+n0,d14:d15
                              ; load ID2:ID N/2+1
   dosetup3 _start_loop1
   ]
   FALIGN
_start_loop1
   loopstart3
       [ ;01
                               ; IB2+ID2
       sod2aaii d14,d10,d10
       sod2ssii d14,d10,d14
                                  ; IB2-ID2
                                 ; IB_N/2+1 + ID_N/2+1
       sod2aaii d15,d11,d11
                                 ; IB_N/2+1 - ID_N/2+1
       sod2ssii d15,d11,d15
       1
       [;02
                                 ; IA_N/2+IC_N/2+(IB_N/2+ID_N/2) = OA_N/2
       sod2aaii d3,d1,d8
                                 ; IA_N/2+IC_N/2-(IB_N/2+ID_N/2) = OB_N/2
       sod2ssii d3,d1,d10
                                 ; IA2+IC2+(IB2+ID2) = OA2
       sod2aaii d10,d8,d1
       sod2ssii d10,d8,d3
                                  ; IA2+IC2-(IB2+ID2) = OB2
       1
       [;03
                                 ; IA_N/2-IC_N/2-j(IB N/2-ID N/2) = OC N/2
       sod2saxx d7,d5,d12
       sod2asxx d7,d5,d14
                                 ; IA N/2-IC N/2+j(IB N/2-ID N/2) = OD N/2
       sod2saxx d14,d12,d5
                                 ; IA2-IC2-j(IB2-ID2) = OC2
                                 ; IA2-IC2+j(IB2-ID2) = OD2
       sod2asxx d14,d12,d7
                               ; save OA1:OA2
       MOVER2.4F d0:d1, (r4)+n0
       MOVE2.4F (r0)+n0,d0:d1 ; load IA1:IA N/2
       1
       [;04
       MOVER2.4F d4:d5, (r4)+n1 ; save OC1:OC2
       MOVE2.4F (r3)+n0,d4:d5
                                 ; load IC1:IC_N/2
       ]
       [;05
       MOVER2.4F d2:d3, (r8)+n0 ; save OB1:OB2
       MOVE2.4F (r0)+n0,d2:d3
                                 ; load IB1:IB_N/2
       ]
       [;06
       sod2aaii d11,d9,d9
                                  ; IA_N/2+1 + IC_N/2+1 + (IB_N/2+1 + ID_N/2+1) = OA_N/2+1
       sod2ssii d11,d9,d11
                                     ; IA N/2+1 + IC N/2+1 - (IB N/2+1 + ID N/2+1) = OB N/2+1
                                  ; OC_N/2+1
       sod2saxx d15,d13,d13
       sod2asxx d15,d13,d15
                                  ; OD_N/2+1
       MOVER2.4F d6:d7,(r8)+n1
                                 ; save OD1:OD2
       MOVE2.4F (r3)+n0,d6:d7
                                  ; load ID1:ID_N/2
       1
       [;07
                                  ; IA1+IC1
       sod2aaii d4,d0,d0
       sod2ssii d4,d0,d4
                                  ; IA1-IC1
       sod2aaii d5,d1,d1
                                  ; IA N/2+IC N/2
       sod2ssii d5,d1,d5
                                  ; IA_N/2-IC_N/2
       MOVER2.4F d8:d9,(r5)+n0
                                 ; save OA N/2:OA N/2+1
```

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NP
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```
MOVE2.4F (r0)+n0,d8:d9
                             ; load IA2:IA N/2+1
    1
    [;08
    sod2aaii d6,d2,d2
                                 ; IB1+ID1
    sod2ssii d6,d2,d6
                                  ; IB1-ID1
    sod2aaii d7,d3,d3
                                  ; IB N/2+ID N/2
    sod2ssii d7,d3,d7
                                 ; IB N/2-ID N/2
    MOVER2.4F d12:d13, (r5) +n1 ; save OC N/2:OC N/2+1
    MOVE2.4F (r3)+n0,d12:d13 ; load IC2:IC_N/2+1
    1
    [;09
    sod2aaii d2,d0,d0
                                 ; IA1+IC1+(IB1+ID1) = OA1
    sod2ssii d2,d0,d2
                                  ; IA1+IC1-(IB1+ID1) = OB1
    sod2asxx d6,d4,d6
                                  ; IA1-IC1-j(IB1-ID1) = OC1
                                 ; IA1-IC1+j(IB1-ID1) = OD1
    MOVER2.4F d10:d11,(r9)+n0 ; save OB N/2:OB N/2+1
    MOVE2.4F (r0)+n0,d10:d11 ; load IB2:IB N/2+1
    ]
    [ ;10
                                 ; IA2+IC2
    sod2aaii d12,d8,d8
                                 ; IA2-IC2
    sod2ssii d12,d8,d12
                                  ; IA_N/2+1 + IC_N/2+1
    sod2aaii d13,d9,d9

      sod2ssii d13,d9,d13
      ; IA_N/2+1 - IC_N/2+1

      MOVER2.4F d14:d15,(r9)+n1
      ; save OD_N/2:OD_N/2+1

      MOVE2.4F (r3)+n0,d14:d15
      ; load ID2:ID_N/2+1

    ]
loopend3
[ ; 01
sod2aaii d14,d10,d10
                           ; IB2+ID2
                           ; IB2-ID2
sod2ssii d14,d10,d14
                           ; IB_N/2+1 + ID_N/2+1
sod2aaii d15,d11,d11
                            ; IB_N/2+1 - ID_N/2+1
sod2ssii d15,d11,d15
move.l (SP-36),b2
                            ; load b2 -> wbdtwiddles
                            ; load b2 -> wbdtwiddles
move.l (SP-36),r2
1
[;02
sod2aaii d3,d1,d8
                             ; IA N/2+IC N/2+(IB N/2+ID N/2) = OA N/2
                             ; IA_N/2+IC_N/2-(IB_N/2+ID_N/2) = OB_N/2
sod2ssii d3,d1,d10
sod2aaii d10,d8,d1
                             ; IA2+IC2+(IB2+ID2) = OA2
sod2ssii d10,d8,d3
                              ; IA2+IC2-(IB2+ID2) = OB2
tfra r0,b5
                               ; data buffer
move.w #4,r14
1
[;03
sod2saxx d7,d5,d12
                             ; IA N/2-IC N/2-j(IB N/2-ID N/2) = OC N/2
sod2asxx d7,d5,d14
                             ; IA N/2-IC N/2+j(IB N/2-ID N/2) = OD N/2
sod2saxx d14,d12,d5
                             ; IA2-IC2-j(IB2-ID2) = OC2
sod2asxx d14,d12,d7
                             ; IA2-IC2=j(IB2-ID2) = OD2
MOVER2.4F d0:d1,(r4)+n0
                               ; save OA1:OA2
dosetup3 _start_loop3_stage2
1
[;04
MOVER2.4F d4:d5,(r4)
                               ; save OC1:OC2
tfra b4,r4
                               ; data buffer
1
[;05
MOVER2.4F d2:d3, (r8)+n0
                               ; save OB1:OB2
move.w (SP-40),d3
                               ; LOG 4 N
1
[;06
sod2aaii d11,d9,d9
                               ; IA N/2+1 + IC N/2+1 + (IB N/2+1 + ID N/2+1) = OA N/2+1
sod2ssii d11,d9,d11
                               ; IA_N/2+1 + IC_N/2+1 - (IB_N/2+1 + ID_N/2+1) = OB_N/2+1
sub #2,d3
                               ; d3 = LOG 4 N - 2
```



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NP
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```
MOVER2.4F d6:d7,(r8)
                           ; save OD1:OD2
   tfra r0,b0
                           ; data buffer = b0,r0
   1
   [;
                         ; OC_N/2+1
   sod2saxx d15,d13,d13
   sod2asxx d15,d13,d15
                         ; OD N/2+1
   MOVER2.4F d8:d9, (r5)+n0
                        ; save OA N/2:OA N/2+1
                         ; data buffer+4*N
   pop r0
   ]
   [;
   MOVER2.4F d12:d13,(r5)
                         ; save OC_N/2:OC_N/2+1
   move.l #($00AA098A),MCTL
                         ; load MCTL: r0,r4,r5: M2 used-Modulo addressing,
                         ;
                                      r1: M0 used-Modulo addressing.
                                      r2: M1 used-Modulo addressing,
                          ;
                                      r3,r6,r7: linear addressing,
                          ;
   ]
   [;
   MOVER2.4F d10:d11,(r9)+n0
                          ; save OB N/2:OB N/2+1
   tfra r3,r5
                          ; data buffer+2*N
   1
   [;
   MOVER2.4F d14:d15,(r9)
                           ; save OD N/2:OD N/2+1
   tfra r1,b1
                           ; load b1 -> wctwiddles
   1
pop MCTL
;
   pop.21 d6:d7
;
;
   rtsd
                          move.l (SP-4),d3
   move.l d3,SR
; 2 STALLS on MCTL
; ENTRY TO THE MAIN
                          ; LOOP (RADIX-4)
                          ;/// n0=N/8, n1=1-N/8, n2=N/16, n3=3N/4, m0=N, m1=N*2, m2=N*8
Γ
   dosetup0 _start_loop2_stage3
   move2.2f (r1),d2
                          ; Load WC1
   1
   [
;//_//
   doen1 d3
                          ; init. lc1
;/// doen1 #1 ; DEBUG
   MOVE2.4F (r0)+,d0:d1
                          ; Load IA1:IC1
   1
   [
   move2.4f (r0)+,d8:d9
                          ; Load IB1:ID1
   move2.4f (r2)+n2,d10:d11
                          ; Load WB1:WD1
   1
   FALIGN
start loop1 stage2:
   loopstart1
      [ ;01
      MAC2ASSAR d1,d2,d0.H,d12
                             ; M1 1.H = IA[re] + IC[re] * WC[re] - IC[im] * WC[im]
                             ; M1_1.L = IA[re] - IC[re] * WC[re] + IC[im] * WC[im]
                             ; M1_2.H = IA[im] + IC[re] * WC[im] + IC[im] * WC[re]
      MAC2AASSI d1,d2,d0.L,d13
                             ; M1 2.L = IA[im] - IC[re] * WC[im] - IC[im] * WC[re]
      doen2 r14
                             ; init. lc2 = 4, 16, ... N/64, N/16
      asra r15
                             ; 2(N/32-1), 2(N/128-1), ...
      1
      [;02
      MAC2ASSAR d8,d10,d12.H,d12 ; M2_1.H = M1_1.H + IB[re] * WB[re] - IB[im] * WB[im]
```

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```
; M2_1.L = M1_1.H - IB[re] * WB[re] + IB[im] * WB[im]
       MAC2AASSI d8,d10,d12.L,d4
                                   ; M2_2.H = M1_1.L + IB[re] * WB[im] + IB[im] * WB[re]
                                    ; M2 2.L = M1 1.L - IB[re] * WB[im] - IB[im] * WB[re]
       MAC2AASSI d8,d10,d13.H,d13
                                   ; M2_3.H = M1_2.H + IB[re] * WB[im] + IB[im] * WB[re]
                                   ; M2_3.L = M1_2.H - IB[re] * WB[im] - IB[im] * WB[re]
       MAC2SAASR d8,d10,d13.L,d5
                                   ; M2_4.H = M1_2.L - IB[re] * WB[re] + IB[im] * WB[im]
                                   ; M2 4.L = M1 2.L + IB[re] * WB[re] - IB[im] * WB[im]
       asl2a r14
       asra r15
                                   ; N/32-1, N/128-1, ..., 7, 1
       1
       FALIGN
_start_loop2_stage2:
       loopstart2
           [;03
                                       ; OA[re] = M2_1.H + ID[re] * WD[re] - ID[im] * WD[im]
           MAC2ASSAR d9,d11,d12
                                       ; OB[re] = M2 1.L - ID[re] * WD[re] + ID[im] * WD[im]
                                       ; OC[re] = M2 2.H - ID[re] * WD[im] - ID[im] * WD[re]
           MAC2SSAAI d9,d11,d4
                                       ; OD[re] = M2_2.L + ID[re] * WD[im] + ID[im] * WD[re]
           MAC2AASSI d9,d11,d13
                                        ; OA[im] = M2 3.H + ID[re] * WD[im] + ID[im] * WD[re]
                                        ; OB[im] = M2_3.L - ID[re] * WD[im] - ID[im] * WD[re]
           MAC2ASSAR d9,d11,d5
                                       ; OC[im] = M2_4.H + ID[re] * WD[re] - ID[im] * WD[im]
                                        ; OD[im] = M2 4.L - ID[re] * WD[re] + ID[im] * WD[im]
           MOVE2.4F (r0)+,d0:d1
                                       ; Load IA2:IC2
           doen3 r15
                                       ; init. lc3 = N/32-1, N/128-1, ...
           1
FALIGN
_start_loop3_stage2:
           loopstart3
               [;04
               MAC2ASSAR d1,d2,d0.H,d6
                                          ; M1 1.H = IA[re] + IC[re] * WC[re] - IC[im] * WC[im]
                                          ; M1_1.L = IA[re] - IC[re] * WC[re] + IC[im] * WC[im]
                                          ; M1_2.H = IA[im] + IC[re] * WC[im] + IC[im] * WC[re]
               MAC2AASSI d1,d2,d0.L,d7
                                          ; M1 2.L = IA[im] - IC[re] * WC[im] - IC[im] * WC[re]
               move2.4f (r0)+,d8:d9
                                          ; Load IB2:ID2
               1
               [;05
                                          ; M2_1.H = M1_1.H + IB[re] * WB[re] - IB[im] * WB[im]
               MAC2ASSAR d8,d10,d6.H,d6
                                          ; M2_1.L = M1_1.H - IB[re] * WB[re] + IB[im] * WB[im]
               MAC2AASSI d8,d10,d6.L,d14 ; M2_2.H = M1_1.L + IB[re] * WB[im] + IB[im] * WB[re]
                                          ; M2 2.L = M1 1.L - IB[re] * WB[im] - IB[im] * WB[re]
                                          ; M2_3.H = M1_2.H + IB[re] * WB[im] + IB[im] * WB[re]
               MAC2AASSI d8,d10,d7.H,d7
                                          ; M2_3.L = M1_2.H - IB[re] * WB[im] - IB[im] * WB[re]
               MAC2SAASR d8,d10,d7.L,d15 ; M2_4.H = M1_2.L - IB[re] * WB[re] + IB[im] * WB[im]
                                          ; M2 4.L = M1 2.L + IB[re] * WB[re] - IB[im] * WB[im]
               MOVE2.4F (r0)+, d0:d1
                                          ; Load IA1:IC1
               ]
               [;06
               MAC2ASSAR d9,d11,d6
                                          ; OA[re] = M2_1.H + ID[re] * WD[re] - ID[im] * WD[im]
                                          ; OB[re] = M2_1.L - ID[re] * WD[re] + ID[im] * WD[im]
               MAC2SSAAI d9,d11,d14
                                          ; OC[re] = M2_2.H - ID[re] * WD[im] - ID[im] * WD[re]
                                          ; OD[re] = M2 2.L + ID[re] * WD[im] + ID[im] * WD[re]
                                          ; OA[im] = M2_3.H + ID[re] * WD[im] + ID[im] * WD[re]
               MAC2AASSI d9,d11,d7
                                          ; OB[im] = M2 3.L - ID[re] * WD[im] - ID[im] * WD[re]
                                          ; OC[im] = M2 4.H + ID[re] * WD[re] - ID[im] * WD[im]
               MAC2ASSAR d9,d11,d15
                                          ; OD[im] = M2_4.L - ID[re] * WD[re] + ID[im] * WD[im]
               move2.4f (r0)+,d8:d9
                                          ; Load IB1:ID1
               1
               [ ;01
               MAC2ASSAR d1,d2,d0.H,d12
                                                 ; M1_1.H = IA[re] + IC[re] * WC[re] - IC[im] * WC[im]
                                                 ; M1 1.L = IA[re] - IC[re] * WC[re] + IC[im] * WC[im]
                                                 ; M1_2.H = IA[im] + IC[re] * WC[im] + IC[im] * WC[re]
               MAC2AASSI d1,d2,d0.L,d13
                                                 ; M1_2.L = IA[im] - IC[re] * WC[im] - IC[im] * WC[re]
```

```
; save OA1:OA2
               MOVERH.4F d12:d13:d6:d7, (r4)+n0
               MOVERL.4F d12:d13:d6:d7,(r5)+n0
                                               ; save OB1:OB2
               1
               [;02
                                            ; M2_1.H = M1_1.H + IB[re] * WB[re] - IB[im] * WB[im]
               MAC2ASSAR d8,d10,d12.H,d12
                                            ; M2<sup>1</sup>.L = M1<sup>1</sup>.H - IB[re] * WB[re] + IB[im] * WB[im]
                                            ; M2 2.H = M1 1.L + IB[re] * WB[im] + IB[im] * WB[re]
               MAC2AASSI d8,d10,d12.L,d4
                                            ; M2 2.L = M1 1.L - IB[re] * WB[im] - IB[im] * WB[re]
                                            ; M2_3.H = M1_2.H + IB[re] * WB[im] + IB[im] * WB[re]
               MAC2AASSI d8,d10,d13.H,d13
                                            ; M2_3.L = M1_2.H - IB[re] * WB[im] - IB[im] * WB[re]
               MAC2SAASR d8,d10,d13.L,d5
                                            ; M2_4.H = M1_2.L - IB[re] * WB[re] + IB[im] * WB[im]
                                            ; M2_4.L = M1_2.L + IB[re] * WB[re] - IB[im] * WB[im]
               MOVERH.4F d4:d5:d14:d15,(r4)+n1 ; save OC1:OC2
               MOVERL.4F d4:d5:d14:d15,(r5)+n1
                                                 ; save OD1:OD2
               1
               [ :03
               MAC2ASSAR d9,d11,d12
                                          ; OA[re] = M2 1.H + ID[re] * WD[re] - ID[im] * WD[im]
                                          ; OB[re] = M2 1.L - ID[re] * WD[re] + ID[im] * WD[im]
               MAC2SSAAI d9,d11,d4
                                          ; OC[re] = M2 2.H - ID[re] * WD[im] - ID[im] * WD[re]
                                          ; OD[re] = M2_2.L + ID[re] * WD[im] + ID[im] * WD[re]
                                          ; OA[im] = M2_3.H + ID[re] * WD[im] + ID[im] * WD[re]
               MAC2AASSI d9,d11,d13
                                          ; OB[im] = M2_3.L - ID[re] * WD[im] - ID[im] * WD[re]
                                          ; OC[im] = M2_4.H + ID[re] * WD[re] - ID[im] * WD[im]
               MAC2ASSAR d9,d11,d5
                                          ; OD[im] = M2_4.L - ID[re] * WD[re] + ID[im] * WD[im]
               MOVE2.4F (r0)+,d0:d1
                                          ; Load IA2:IC2
               1
loopend3
           [;04
           MAC2ASSAR d1,d2,d0.H,d6
                                      ; M1_1.H = IA[re] + IC[re] * WC[re] - IC[im] * WC[im]
                                      ; M1_1.L = IA[re] - IC[re] * WC[re] + IC[im] * WC[im]
           MAC2AASSI d1,d2,d0.L,d7
                                      ; M1_2.H = IA[im] + IC[re] * WC[im] + IC[im] * WC[re]
                                      ; M1_2.L = IA[im] - IC[re] * WC[im] - IC[im] * WC[re]
           move2.4f (r0)+,d8:d9
                                      ; Load IB2:ID2
           addl2a n2,r1
           1
           [;05
                                      ; M2_1.H = M1_1.H + IB[re] * WB[re] - IB[im] * WB[im]
           MAC2ASSAR d8,d10,d6.H,d6
                                      ; M2_1.L = M1_1.H - IB[re] * WB[re] + IB[im] * WB[im]
           MAC2AASSI d8,d10,d6.L,d14
                                      ; M2_2.H = M1_1.L + IB[re] * WB[im] + IB[im] * WB[re]
                                      ; M2 2.L = M1 1.L - IB[re] * WB[im] - IB[im] * WB[re]
                                      ; M2_3.H = M1_2.H + IB[re] * WB[im] + IB[im] * WB[re]
           MAC2AASSI d8,d10,d7.H,d7
                                      ; M2_3.L = M1_2.H - IB[re] * WB[im] - IB[im] * WB[re]
           MAC2SAASR d8,d10,d7.L,d15 ; M2_4.H = M1_2.L - IB[re] * WB[re] + IB[im] * WB[im]
                                      ; M2 4.L = M1 2.L + IB[re] * WB[re] - IB[im] * WB[im]
           MOVE2.4F (r0)+,d0:d1
                                      ; Load IA1:IC1
           move2.2f (r1),d2
                                      ; Load WC1
           1
           [ :06
                                      ; OA[re] = M2_1.H + ID[re] * WD[re] - ID[im] * WD[im]
           MAC2ASSAR d9,d11,d6
                                      ; OB[re] = M2_1.L - ID[re] * WD[re] + ID[im] * WD[im]
                                      ; OC[re] = M2 2.H - ID[re] * WD[im] - ID[im] * WD[re]
           MAC2SSAAI d9,d11,d14
                                      ; OD[re] = M2_2.L + ID[re] * WD[im] + ID[im] * WD[re]
           MAC2AASSI d9,d11,d7
                                      ; OA[im] = M2 3.H + ID[re] * WD[im] + ID[im] * WD[re]
                                      ; OB[im] = M2 3.L - ID[re] * WD[im] - ID[im] * WD[re]
           MAC2ASSAR d9,d11,d15
                                      ; OC[im] = M2_4.H + ID[re] * WD[re] - ID[im] * WD[im]
                                      ; OD[im] = M2_4.L - ID[re] * WD[re] + ID[im] * WD[im]
           move2.4f (r0)+,d8:d9
                                      ; Load IB1:ID1
           move2.4f (r2)+n2,d10:d11
                                      ; Load WB1:WD1
           1
           [;01
                                             ; M1_1.H = IA[re] + IC[re] * WC[re] - IC[im] * WC[im]
           MAC2ASSAR d1,d2,d0.H,d12
                                             ; M1_1.L = IA[re] - IC[re] * WC[re] + IC[im] * WC[im]
```

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```
MAC2AASST d1.d2.d0.t.d13
                                           ; M1_2.H = IA[im] + IC[re] * WC[im] + IC[im] * WC[re]
                                           ; M1_2.L = IA[im] - IC[re] * WC[im] - IC[im] * WC[re]
           MOVERH.4F d12:d13:d6:d7, (r4)+n0
                                           ; save OA1:OA2
           MOVERL.4F d12:d13:d6:d7,(r5)+n0
                                           ; save OB1:OB2
           1
           [:02
           MAC2ASSAR d8,d10,d12.H,d12
                                      ; M2 1.H = M1 1.H + IB[re] * WB[re] - IB[im] * WB[im]
                                       ; M2 1.L = M1 1.H - IB[re] * WB[re] + IB[im] * WB[im]
           MAC2AASSI d8,d10,d12.L,d4
                                       ; M2_2.H = M1_1.L + IB[re] * WB[im] + IB[im] * WB[re]
                                       ; M2_2.L = M1_1.L - IB[re] * WB[im] - IB[im] * WB[re]
                                       ; M2_3.H = M1_2.H + IB[re] * WB[im] + IB[im] * WB[re]
           MAC2AASSI d8,d10,d13.H,d13
                                       ; M2_3.L = M1_2.H - IB[re] * WB[im] - IB[im] * WB[re]
           MAC2SAASR d8,d10,d13.L,d5
                                       ; M2 4.H = M1 2.L - IB[re] * WB[re] + IB[im] * WB[im]
                                       ; M2 4.L = M1 2.L + IB[re] * WB[re] - IB[im] * WB[im]
           MOVERH.4F d4:d5:d14:d15,(r4)+n1 ; save OC1:OC2
           MOVERL.4F d4:d5:d14:d15,(r5)+n1 ; save OD1:OD2
           1
       loopend2
       [
       asra n2
       tfra b2,r2
       1
       [
       asra n2
       addl2a n3,r4
                                    ; r4+OFFSET
       ]
       [
                                    ; r5+OFFSET
       addl2a n3,r5
       move2.4f (r2)+n2,d10:d11
                                    ; Load WB1:WD1
       1
   loopend1
;//// the commented instructions in the following 2 VELS were computed in previous loop
[ ;01
                              ; M1 1.H = IA[re] + IC[re] * WC[re] - IC[im] * WC[im]
    MAC2ASSAR d1,d2,d0.H,d12
;
                              ; M1 1.L = IA[re] - IC[re] * WC[re] + IC[im] * WC[im]
;
                              ; M1_2.H = IA[im] + IC[re] * WC[im] + IC[im] * WC[re]
    MAC2AASSI d1,d2,d0.L,d13
;
                              ; M1_2.L = IA[im] - IC[re] * WC[im] - IC[im] * WC[re]
;
   move2.4f (r1)+,d2:d3
                              ; Load WC1:WC2
   ]
   [;02
    MAC2ASSAR d8,d10,d12.H,d12 ; M2_1.H = M1_1.H + IB[re] * WB[re] - IB[im] * WB[im]
;
                               ; M2 1.L = M1 1.H - IB[re] * WB[re] + IB[im] * WB[im]
;
    MAC2AASSI
              d8,d10,d12.L,d4
                              ; M2 2.H = M1 1.L + IB[re] * WB[im] + IB[im] * WB[re]
;
                              ; M2 2.L = M1 1.L - IB[re] * WB[im] - IB[im] * WB[re]
;
    MAC2AASSI d8,d10,d13.H,d13 ; M2_3.H = M1_2.H + IB[re] * WB[im] + IB[im] * WB[re]
;
                              ; M2_3.L = M1_2.H - IB[re] * WB[im] - IB[im] * WB[re]
;
    MAC2SAASR d8,d10,d13.L,d5
                              ; M2_4.H = M1_2.L - IB[re] * WB[re] + IB[im] * WB[im]
;
                              ; M2_4.L = M1_2.L + IB[re] * WB[re] - IB[im] * WB[im]
;
   asra r14
                              ; N/8
   MOVE2.4F (r0)+,d0:d1
                              ; Load IA2:IC2
   ]
   [;03
   MAC2ASSAR d9,d11,d12
                             ; OA[re] = M2 1.H + ID[re] * WD[re] - ID[im] * WD[im]
                             ; OB[re] = M2_1.L - ID[re] * WD[re] + ID[im] * WD[im]
                             ; OC[re] = M2_2.H - ID[re] * WD[im] - ID[im] * WD[re]
   MAC2SSAAI d9,d11,d4
                             ; OD[re] = M2_2.L + ID[re] * WD[im] + ID[im] * WD[re]
   MAC2AASSI d9,d11,d13
                             ; OA[im] = M2_3.H + ID[re] * WD[im] + ID[im] * WD[re]
                              ; OB[im] = M2_3.L - ID[re] * WD[im] - ID[im] * WD[re]
                              ; OC[im] = M2_4.H + ID[re] * WD[re] - ID[im] * WD[im]
   MAC2ASSAR d9,d11,d5
                              ; OD[im] = M2_4.L - ID[re] * WD[re] + ID[im] * WD[im]
   deca r14
                              ; N/8-1
```

```
1
[;04
                              ; M1_1.H = IA[re] + IC[re] * WC[re] - IC[im] * WC[im]
   MAC2ASSAR d1,d3,d0.H,d6
                              ; M1_1.L = IA[re] - IC[re] * WC[re] + IC[im] * WC[im]
                              ; M1 2.H = IA[im] + IC[re] * WC[im] + IC[im] * WC[re]
   MAC2AASSI d1,d3,d0.L,d7
                              ; M1 2.L = IA[im] - IC[re] * WC[im] - IC[im] * WC[re]
   move2.4f (r0)+,d8:d9
                              ; Load IB2:ID2
   move2.4f (r1)+,d2:d3
                              ; Load WC1:WC2
   1
   [;05
   MAC2ASSAR d8,d10,d6.H,d6
                                ; M2 1.H = M1 1.H + IB[re] * WB[re] - IB[im] * WB[im]
                                ; M2 1.L = M1 1.H - IB[re] * WB[re] + IB[im] * WB[im]
                                ; M2_2.H = M1_1.L + IB[re] * WB[im] + IB[im] * WB[re]
   MAC2AASSI d8,d10,d6.L,d14
                                ; M2 2.L = M1 1.L - IB[re] * WB[im] - IB[im] * WB[re]
                                ; M2 3.H = M1 2.H + IB[re] * WB[im] + IB[im] * WB[re]
   MAC2AASSI d8,d10,d7.H,d7
                                ; M2_3.L = M1_2.H - IB[re] * WB[im] - IB[im] * WB[re]
                                ; M2 4.H = M1 2.L - IB[re] * WB[re] + IB[im] * WB[im]
   MAC2SAASR d8,d10,d7.L,d15
                                ; M2_4.L = M1_2.L + IB[re] * WB[re] - IB[im] * WB[im]
   doen0 r14
                                ; init. lc2 = N/8-1
   MOVE2.4F (r0)+,d0:d1
                                ; Load IA1:IC1
   1
   [ ;06
                              ; OA[re] = M2 1.H + ID[re] * WD[re] - ID[im] * WD[im]
   MAC2ASSAR d9,d11,d6
                              ; OB[re] = M2 1.L - ID[re] * WD[re] + ID[im] * WD[im]
   MAC2SSAAI d9,d11,d14
                              ; OC[re] = M2_2.H - ID[re] * WD[im] - ID[im] * WD[re]
                              ; OD[re] = M2 2.L + ID[re] * WD[im] + ID[im] * WD[re]
                              ; OA[im] = M2_3.H + ID[re] * WD[im] + ID[im] * WD[re]
   MAC2AASSI d9.d11.d7
                              ; OB[im] = M2_3.L - ID[re] * WD[im] - ID[im] * WD[re]
                              ; OC[im] = M2_4.H + ID[re] * WD[re] - ID[im] * WD[im]
   MAC2ASSAR d9,d11,d15
                              ; OD[im] = M2_4.L - ID[re] * WD[re] + ID[im] * WD[im]
   move2.4f (r0)+,d8:d9
                              ; Load IB1:ID1
   move2.4f (r2)+,d10:d11
                             ; Load WB1:WD1
   1
   FALIGN
_start_loop2_stage3:
   loopstart0
       [ ;01
       MAC2ASSAR d1,d2,d0.H,d12
                                  ; M1 1.H = IA[re] + IC[re] * WC[re] - IC[im] * WC[im]
                                  ; M1 1.L = IA[re] - IC[re] * WC[re] + IC[im] * WC[im]
                                  ; M1_2.H = IA[im] + IC[re] * WC[im] + IC[im] * WC[re]
       MAC2AASSI d1,d2,d0.L,d13
                                  ; M1_2.L = IA[im] - IC[re] * WC[im] - IC[im] * WC[re]
       MOVERH.4F d12:d13:d6:d7,(r4)+n0
                                       ; save OA1:OA2
       MOVERL.4F d12:d13:d6:d7,(r5)+n0
                                       ; save OB1:OB2
       1
       [;02
       MAC2ASSAR d8,d10,d12.H,d12
                                   ; M2_1.H = M1_1.H + IB[re] * WB[re] - IB[im] * WB[im]
                                    ; M2_1.L = M1_1.H - IB[re] * WB[re] + IB[im] * WB[im]
                                    ; M2_2.H = M1_1.L + IB[re] * WB[im] + IB[im] * WB[re]
       MAC2AASSI d8,d10,d12.L,d4
                                    ; M2_2.L = M1_1.L - IB[re] * WB[im] - IB[im] * WB[re]
                                    ; M2 3.H = M1 2.H + IB[re] * WB[im] + IB[im] * WB[re]
       MAC2AASSI d8,d10,d13.H,d13
                                    ; M2_3.L = M1_2.H - IB[re] * WB[im] - IB[im] * WB[re]
                                    ; M2_4.H = M1_2.L - IB[re] * WB[re] + IB[im] * WB[im]
       MAC2SAASR d8,d10,d13.L,d5
                                    ; M2 4.L = M1 2.L + IB[re] * WB[re] - IB[im] * WB[im]
       MOVERH.4F d4:d5:d14:d15,(r4)+n1 ; save OC1:OC2
       MOVERL.4F d4:d5:d14:d15,(r5)+n1 ; save OD1:OD2
       ]
       [;03
       MAC2ASSAR d9,d11,d12
                                  ; OA[re] = M2_1.H + ID[re] * WD[re] - ID[im] * WD[im]
                                  ; OB[re] = M2 1.L - ID[re] * WD[re] + ID[im] * WD[im]
                                  ; OC[re] = M2_2.H - ID[re] * WD[im] - ID[im] * WD[re]
       MAC2SSAAI d9,d11,d4
                                  ; OD[re] = M2 2.L + ID[re] * WD[im] + ID[im] * WD[re]
```

: Load WB2:WD2

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move2.4f (r2)+,d10:d11

```
NP
```

```
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```

```
; OA[im] = M2 3.H + ID[re] * WD[im] + ID[im] * WD[re]
      MAC2AASST d9.d11.d13
                               ; OB[im] = M2_3.L - ID[re] * WD[im] - ID[im] * WD[re]
      MAC2ASSAR d9,d11,d5
                               ; OC[im] = M2 4.H + ID[re] * WD[re] - ID[im] * WD[im]
                               ; OD[im] = M2 4.L - ID[re] * WD[re] + ID[im] * WD[im]
      MOVE2.4F (r0)+,d0:d1
                               ; Load IA2:IC2
      move2.4f (r2)+,d10:d11
                               ; Load WB2:WD2
      1
[;04
      MAC2ASSAR d1,d3,d0.H,d6
                               ; M1_1.H = IA[re] + IC[re] * WC[re] - IC[im] * WC[im]
                               ; M1_1.L = IA[re] - IC[re] * WC[re] + IC[im] * WC[im]
      MAC2AASSI d1,d3,d0.L,d7
                               ; M1_2.H = IA[im] + IC[re] * WC[im] + IC[im] * WC[re]
                               ; M1 2.L = IA[im] - IC[re] * WC[im] - IC[im] * WC[re]
      move2.4f (r0)+,d8:d9
                               ; Load IB2:ID2
      move2.4f (r1)+,d2:d3
                               ; Load WC1:WC2
      1
      [;05
      MAC2ASSAR d8,d10,d6.H,d6
                               ; M2_1.H = M1_1.H + IB[re] * WB[re] - IB[im] * WB[im]
                               ; M2 1.L = M1 1.H - IB[re] * WB[re] + IB[im] * WB[im]
                               ; M2_2.H = M1_1.L + IB[re] * WB[im] + IB[im] * WB[re]
      MAC2AASSI d8,d10,d6.L,d14
                               ; M2_2.L = M1_1.L - IB[re] * WB[im] - IB[im] * WB[re]
      MAC2AASSI d8,d10,d7.H,d7
                               ; M2 3.H = M1 2.H + IB[re] * WB[im] + IB[im] * WB[re]
                               ; M2_3.L = M1_2.H - IB[re] * WB[im] - IB[im] * WB[re]
      MAC2SAASR d8,d10,d7.L,d15 ; M2_4.H = M1_2.L - IB[re] * WB[re] + IB[im] * WB[im]
                               ; M2 4.L = M1 2.L + IB[re] * WB[re] - IB[im] * WB[im]
      MOVE2.4F (r0)+,d0:d1
                               ; Load IA1:IC1
      1
       [ ;06
      MAC2ASSAR d9,d11,d6
                               ; OA[re] = M2_1.H + ID[re] * WD[re] - ID[im] * WD[im]
                               ; OB[re] = M2_1.L - ID[re] * WD[re] + ID[im] * WD[im]
                               ; OC[re] = M2 2.H - ID[re] * WD[im] - ID[im] * WD[re]
      MAC2SSAAI d9,d11,d14
                               ; OD[re] = M2_2.L + ID[re] * WD[im] + ID[im] * WD[re]
                               ; OA[im] = M2_3.H + ID[re] * WD[im] + ID[im] * WD[re]
      MAC2AASSI d9,d11,d7
                               ; OB[im] = M2 3.L - ID[re] * WD[im] - ID[im] * WD[re]
                               ; OC[im] = M2 4.H + ID[re] * WD[re] - ID[im] * WD[im]
      MAC2ASSAR d9,d11,d15
                               ; OD[im] = M2 4.L - ID[re] * WD[re] + ID[im] * WD[im]
      move2.4f (r0)+,d8:d9
                              ; Load IB1:ID1
      move2.4f (r2)+,d10:d11
                               ; Load WB1:WD1
      1
   loopend0
   [
   MOVERH.4F d12:d13:d6:d7,(r4)+n0 ; save OA1:OA2
   MOVERL.4F d12:d13:d6:d7,(r5)+n0 ; save OB1:OB2
   1
   Г
   MOVERH.4F d4:d5:d14:d15,(r4)
                               ; save OC1:OC2
   pop MCTL
   1
   ſ
   MOVERL.4F d4:d5:d14:d15,(r5)
                               ; save OD1:OD2
   pop.21 d6:d7
   1
   pop.21 r6:r7
global F sc3850 fft radix4 complex 16x16 asm end
F_sc3850_fft_radix4_complex_16x16_asm_end:
                                           ;// finish to count cycles
end benchmark:
   rtsd
                           move.l (SP-4),d3
   move.l d3,SR
  -----TEST LOOP-----
;
   FNDSFC
```



4.6 Fixed-Point Arithmetic and Bit Accuracy

The SC3850 core architecture uses the fixed-point representation. The finite precision for the Radix-4 DIT FFT is limited by the following parameters:

- Number of bits that are available in the input and output data
- Operations of finite arithmetic calculations (addition, subtraction, and multiplication with rounding)

The data of Radix-4 DIT butterfly can be scaled down by a factor of 1, 2, or 4 and is equivalent to an arithmetic right shift by 0, 1, or 2 bits to avoid the possibility of overflow. In the zero shift case, the final significant bits are 16-bit Q15 data, which uses all 16 bits at the risk of overflowing. If the output is scaled down by 4, the final significant bits are 14-bit Q13 data, which avoids the overflowing of the output data. In summary, the output of each butterfly should have $14 \sim 16$ -bit accuracy for fixed-scaling method (Q13-Q15 in Q format).

4.7 Implementation of Radix-4 DIT IFFTs

A common method to implement the Inverse Fast Fourier Transform (IFFT) is to change the sign of the twiddle factors and use the FFT subroutine. This method needs additional memory to store the twiddle factors. Therefore, it is not efficient for reduction of memory requirement.

If the complex conjugate of FFT (Equation 3) is considered, the equation is defined by Equation 21.

$$\begin{aligned} \mathbf{x}^{*}(\mathbf{n}) &= \frac{1}{N} \sum_{\substack{k=0 \\ N-1}} \left[\mathbf{X}(k) \exp\left(j\frac{2\pi nk}{N}\right) \right]^{*} \\ &= \frac{1}{N} \sum_{\substack{k=0 \\ N-1}} \mathbf{X}^{*}(k) \exp\left((-j)\frac{2\pi nk}{N}\right) \\ &= \frac{1}{N} \sum_{\substack{N=1 \\ N-1}} \mathbf{X}^{*}(k) \left\{ \cos\left(\frac{2\pi nk}{N}\right) - j\sin\left(\frac{2\pi nk}{N}\right) \right\} \end{aligned}$$
Eqn. 21



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The asterisk indicates the complex conjugate. In this form, there is no need to have additional memory for the twiddle factors since the twiddle factor is the same as the FFT. FFT is applied on the complex conjugate of X(k) (with appropriate scaling) to implement IFFT. The complex conjugate of the resulting time signal x(n) is the output of the IFFT. By having routines for both the FFT and the IFFT, there is no need to have the negative signed twiddle factors.

If the FFT is implemented with scaling as described in Section 4.1, the resulting values in the frequency domain are (1/N)X(k). For IFFT, no scaling should be applied in order to get back the original data.

The advantages of this implementation are summarized as follows:

- The same twiddle factors can be used for both the FFT and the IFFT. There is no need to save additional twiddle factors.
- FFT routine uses the 1/N factor and the IFFT routine does not. Therefore, there is more flexibility regarding scaling.

Equation 22 describes the Radix-4 DIT butterfly calculation of the IFFT:

 $\begin{aligned} Ar' &= Ar + (Cr \times Wcr + Ci \times Wci) + (Br \times Wbr + Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Ai' &= Ai - (Cr \times Wci - Ci \times Wcr) - (Br \times Wbi - Bi \times Wbr) - (Dr \times Wdi - Di \times Wdr) \\ Br' &= Ar + (Cr \times Wcr + Ci \times Wci) - (Br \times Wbr + Bi \times Wbi) - (Dr \times Wdr + Di \times Wdi) \\ Bi' &= Ai - (Cr \times Wci - Ci \times Wcr) + (Br \times Wbi - Bi \times Wbr) + (Dr \times Wdi - Di \times Wdr) \\ Cr' &= Ar - (Cr \times Wcr + Ci \times Wci) + (Br \times Wbi - Bi \times Wbr) - (Dr \times Wdi - Di \times Wdr) \\ Ci' &= Ai + (Cr \times Wci - Ci \times Wcr) + (Br \times Wbr + Bi \times Wbi) - (Dr \times Wdr + Di \times Wdi) \\ Dr' &= Ar - (Cr \times Wcr + Ci \times Wci) - (Br \times Wbr + Bi \times Wbr) + (Dr \times Wdi - Di \times Wdr) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi - Bi \times Wbr) + (Dr \times Wdi - Di \times Wdr) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi - Bi \times Wbr) + (Dr \times Wdi - Di \times Wdr) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi - Bi \times Wbr) + (Dr \times Wdi - Di \times Wdr) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi - Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi - Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi - Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi - Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi - Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi + Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi + Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi + Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi + Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi + Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) - (Br \times Wbi + Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) + (Br \times Wbi + Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) + (Br \times Wbi + Bi \times Wbi) + (Dr \times Wdr + Di \times Wdi) \\ Di' &= Ai + (Cr \times Wci - Ci \times Wci) + (Dr \times Wbi + Bi \times Wbi) \\ Di' &= Ai + (Cr \times Wci + Di \times Wci) \\ Di' &= Ai + (C$

The differences between the FFT (Equation 20) and the IFFT (Equation 22) are the signs between operands. So the IFFT can be easily implemented by modifying the FFT subroutine.



5 Experimental Results

5.1 Test Vectors

Test vectors of length 256, 1024, 4096 are used to test the functionality of the corresponding FFT/IFFT SC3850 kernels. The 256-point test vector is shown in Figure 19.



Figure 19. Test Vector for 256-point FFT



Experimental Results

5.2 Performance

5.2.1 FFT SNR Results

The FFT results obtained from the SC3850 core are compared with the reference, which is shown in Figure 20 for the test vector of length 256. As the figure indicates, the SC3850 core and the reference results are virtually identical.



Figure 20. 256-Point FFT Results from Matlab and the SC3850 Core

Signal-to-noise (SNR) was calculated between the reference output and the SC3850 core output. Table 10 shows the SNR results for the different point FFTs with fixed scaling. The SNR for FFT is computed using: $SNR(dB) = 10*log(total_signal_power/total_noise_power)$. Given two complex signals, A and B, where signal A is the reference signal and signal B is the signal that is corrupted by noise (fixed-point), the SNR is found using $10*log((sum(A_{Re}^2)+sum(A_{Im}^2))/((sum(A_{Re}-B_{Re})^2+sum(A_{Im}-B_{Im})^2)))$.

N =	SNR (dB)
256	75.5
1024	75.1
4096	74.8

Table 10. SNR fo	or Different Point FFTs
------------------	-------------------------



5.2.2 Cycle Count Measurements

The real-time cycle counts for the implemented Radix-4 DIT FFT/IFFT kernel are measured using the SC3850 cycle accurate simulator. CodeWarrior Integrated Development Environment (IDE) R4.0 is used to compiler the FFT/IFFT kernels and the corresponding test harness.

The cycle count and memory usage for Radix-4 DIT FFT/IFFT kernel (256, 1024, and 4096 points) are summarized in Table 11. The conversion of the number of cycles into microseconds is also listed under an assumption that the SC3850 core frequency is 1GHz.

Kernel	Cycle Count	Time (μs)	Memory Usage (Bytes)
256-FFT	792	0.79	950
1024-FFT	3773	3.77	950
4096-FFT	17986	17.99	950
256-IFFT	789	0.79	936
1024-IFFT	3770	3.77	936
4096-IFFT	17983	17.98	936

Table 11. C	vcle Count and	Memory Us	sage of FFT	/IFFT Kernels
	,			



6 Conclusions

The purpose of this application note is to develop Radix-4 DIT FFT algorithm implemented on the SC3850 core architecture. The implementation aspects of Radix-4 DIT FFT are based on the following SC3850 features:

- VLES execution model, which utilizes maximum parallelism by allowing multiple address generation and data arithmetic logic units to execute multiple instructions in a single cycle
- Special SIMD instructions, such as SOD2ffcc, MAC2ffggR, and MAC2ffggI
- MOVER instructions with scaling, rounding, and limiting to avoid overflowing
- Memory access with multiple data, such as MOVE2.4F, MOVE2.2f, MOVERH.4f, and MOVERL.4F.
- Bit-reversed (reverse-carry) addressing mode to support the bit-reversed order of data

The SC3850 fixed-point code incurs very little loss of accuracy compared to the floating-point Matlab results. The SC3850 core performance shows that the execution time of 1024 points FFT is 3.77 μ s assuming the core frequency of 1 GHz. The data memory required to store input samples is 4N bytes, 4N bytes for output data, and 3N bytes for twiddle factors for N-point FFT. In addition, at each stage of the FFT algorithm, the output data can be scaled down by 1, 2, or 4 to avoid overflowing. The scaling factor is a parameter to the FFT kernel.

The powerful architecture and instruction set of the SC3850 core permits flexible and compact coding of the algorithms in assembly language. The optimization of Radix-4 DIT FFT is done by taking special advantage of StarCore parallel computation capabilities, such as SIMD, parallel computing, and software pipelining.

7 References

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References

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